Applied Differential Equations – Mth 256

Archive – Summer 1992 Files

Jan 11, 2001

This archive contains the numerous short tests (15 to 30 minutes) from Mth 256 Summer 1992. The original test instructions, headers and formatting have not been preserved.

Contents

1 Test 1

Problem 1. Solve the ordinary differential equation

$$2xy\frac{dy}{dx} = y^2 + 1.$$

Problem 2. Solve the initial value problem

$$\frac{dy}{dx} = \frac{x^3 + y^3}{xy^2}, \qquad y(1) = 2.$$

Problem 3. Solve the initial value problem

$$\frac{dy}{dx} + y\cos(x) = \cos(x), \qquad y(0) = 3.$$

Problem 4. Make the substitution $z = x + y$ to solve the ordinary differential equation

$$\frac{dy}{dx} = \big(x + y + 2\big)\big(x + y\big).$$

Problem 5. Solve the initial value problem

$$\big(3x^2y^2 - 2xy^3 + 2x + 1\big)\,dx + \big(2x^3y - 3x^2y^2 - 3y^2 + 1\big)\,dy = 0, \qquad y(0) = 2.$$

Problem 6. A tank initially contains 100 L of brine of concentration 2.1 g/L salt. Brine of concentration 0.6 g/L runs into the tank at 3.0 L/min and the well-mixed solution is drained off at 2.0 L/min. Find the concentration of salt in the tank at the moment that the tank contains 200 L brine.

2 Test 1 Make–Up

Two problems on the make–up test were identical to two problems on the regular test and have been omitted here.

Problem 7. Solve the ordinary differential equation

$$y\frac{dy}{dx} = x\,e^{x^2+y^2}.$$

Problem 8. Solve the initial value problem

$$\frac{dy}{dx} = \frac{x^3 + y^3}{xy^2}, \qquad y(1) = 2.$$

Problem 9. Solve the initial value problem

$$\frac{dy}{dx} + y\cos(x) = \cos(x), \qquad y(0) = 3.$$

Problem 10. Make the substitution $z = x + y$ to solve the ordinary differential equation

$$\frac{dy}{dx} = \frac{\big(x + y - 1\big)\big(x + y\big)}{2x + 2y + 1}$$

Problem 11. Solve the initial value problem

$$\big(3x^2y^2 - 2xy^3 - 2x - 1\big)\,dx + \big(2x^3y - 3x^2y^2 - 8y^3 - y + 1\big)\,dy = 0, \qquad y(0) = 2.$$

Problem 12. A tank initially contains 100 L of brine of concentration 0.6 g/L salt. Brine of concentration 2.1 g/L runs into the tank at 6.0 L/min and the well-mixed solution is drained off at 4.0 L/min. Find the concentration of salt in the tank at the moment that the tank contains 220 L brine.

3 Test 2

Problem 13. Consider the 1-parameter family of hyperbolas and ellipses given by

$$x^2 - \alpha y^2 = 1 \qquad\qquad \alpha \text{ a constant (the parameter).}$$

Find the 1-parameter family of orthogonal trajectories.

Problem 14. A thermometer is brought into a certain room. The room has temperature $A = 25°$ C. If T is the temperature displayed by the thermometer then according to NEWTON

$$\frac{dT}{dt} = -k(T - A)$$

where k is a constant. After being in the room for 10 seconds the thermometer reads 21.4° C. An additional 20 seconds later it reads 23.4° C. What was the initial reading on the thermometer at the time that it was first brought into the room?

Problem 15. Use the substitution $y = x^2 w$ to solve the ordinary differential equation

$$\frac{dy}{dx} = \frac{2y^2 + x^3}{xy}.$$

4 Test 3

Find the general solution in REAL FORM for each of the following eight problems.

Problem 16.

$$y'' - 2y' - 48y = 0$$

Problem 17.

$$y'' - 2y' = 0$$

Problem 18.

$$y'' + 9y = 0$$

Problem 19.

$$y'' - 9y = 0$$

Problem 20.

$$y'' - 6y' + 9y = 0$$

Problem 21.

$$y'' + 4y' + 20y = 0$$

Problem 22.

$$y^{(4)} - y^{(3)} = 0$$

Problem 23.

$$y^{(4)} + 8y^{(2)} + 16y = 0$$

Problem 24. The differential equation

$$x^2y'' - 6y = x^3 + 2x$$

has complementary solution

$$c_1x^3 + c_2x^{-2}.$$

Use variation of parameters to find a particular solution.

Problem 25. The function $y_1 = x$ is a solution of the ode

$$(1 - x)y'' + xy' - y = 0.$$

Use reduction of order to find another solution y_2 such that $\{y_1, y_2\}$ is a fundamental solution set.

5 Test 4

For the first 4 problems find the general solution in REAL FORM.

Problem 26.

$$x^2y'' - 12y = 0.$$

Problem 27.

$$x^2y'' + 5xy' + 13y = 0.$$

Problem 28.

$$x^2y'' - 3xy' + 4y = 0.$$

Problem 29.

$$x^2y'' + 4xy' = 0.$$

For the next three problems use the method of undetermined coefficients (judicious guessing) to find a particular solution.

Problem 30.

$$2y'' + 7y' - 5y = -5x^3 + 21x^2 + 7x + 12.$$

Problem 31.

$$y'' + 2y' - 3y = (8x - 2)\,e^x.$$

Problem 32.

$$y'' - y = x\,e^x\,\cos(x).$$

Problem 33. Find the general solution of the ordinary differential equation

$$\frac{d^2y}{dx^2} = \frac{2y}{y^2 + 1}\left(\frac{dy}{dx}\right)^2.$$

6 Test 5

Problem 34. A certain spring stretches by 0.20 meter when a mass of 20.0 kilograms is suspended from it. (A) Assuming that the acceleration of gravity is 9.8 meters/sec^2 calculate the spring constant k in Newtons/meter. (B) What is the natural frequency in Hertz (cycles/sec)? Suppose now that a periodic force with downward component $80\sin(8t)$ Newtons is applied to the mass. (C) Assuming that the mass starts at rest at equilibrium, find its motion at all subsequent times.

Problem 35. An RLC series circuit has an electromotive force $E(t) = E_0 cos(\omega t)$. The equation for the current is then

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I = -E_0\omega\ \sin(\omega t).$$

The solution to this equation is the sum of an exponentially decreasing complementary solution and a unique periodic solution of the form

$$I(t) = \frac{1}{N}\Im\big(E_0 e^{i(\omega t+\phi)}\big) = \frac{1}{N}E_0\sin(\omega t+\phi)$$

where $\Im$ indicates the imaginary part and $N > 0$ is a constant. Compute N in terms of R, L, C and ω.

Problem 36. Compute the LAPLACE transform

$$\mathcal{L}\left\{e^{2t}\cos(3t)\right\}.$$

Problem 37. If

$$\mathcal{L}\left\{f(t)\right\} = \frac{s^3}{s^4 - s + 2}$$

compute the LAPLACE transform

$$\mathcal{L}\left\{e^{-2t}f(t)\right\}.$$

Problem 38. If

$$f(t) = \begin{cases} 3t, & 0 \le t \le 2 \\ 6, & 2 \le t. \end{cases}$$

compute the LAPLACE transform

$$\mathcal{L}\left\{f(t)\right\} = \int_0^\infty e^{-st}f(t)\,dt.$$

7 Test 6

The original test included a table of LAPLACE transforms. That table has not been included in this archive.

Problem 39.

Compute and simplify

$$\mathcal{L}\left\{e^{2t}\cos 3t\right\}.$$

Problem 40.

Compute and simplify

$$\mathcal{L}\left\{\cos^2 2t\right\}.$$

Problem 41.

Compute and simplify

$$\mathcal{L}\left\{e^t \cos t \sin t\right\}.$$

Problem 42.

Compute and simplify

$$\mathcal{L}\left\{\frac{e^t - e^{-t}}{t}\right\}.$$

Problem 43.

Compute and simplify

$$\mathcal{L}\left\{t \sin 2t\right\}.$$

Problem 44.

Compute and simplify

$$\mathcal{L}^{-1}\left\{\frac{s-1}{s^2}\right\}.$$

Problem 45.

Compute and simplify

$$\mathcal{L}^{-1}\left\{\frac{s^2+s+6}{(s+1)^2(s-1)}\right\}.$$

Problem 46.

Compute and simplify

$$\mathcal{L}^{-1}\left\{\frac{4(s+1)}{s(s^2+4)}\right\}.$$

Problem 47. Find the LAPLACE transform of the solution to the initial value problem

$$2y'' - 3y' + 2y = te^t, \qquad y(0) = -1, \quad y'(0) = 2.$$

Problem 48. If

$$y(t) + \int_0^t y(r)\, dr = 1$$

for each t, use the LAPLACE transform to find $y(t)$.

8 Contact Information

The contact information below is accurate as of Jan 11, 2001.

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