Applied Differential Equations – Mth 256

Archive – Summer 1993 Files

Jan 11, 2001

This archive contains the sample problems and tests from Mth 256 Summer 1993. The original test instructions, headers and formatting have not been preserved.

Contents

1 Sample Problem Set 1

Problem 1. Solve the initial value problem

$$\frac{dy}{dx} = y(1+x^2) - (1+x^2), \quad y(0) = 2.$$

Problem 2. Solve the initial value problem

$$\frac{dy}{dx} = \frac{4x^2 + y^2}{2xy}, \quad y(2) = 2.$$

Problem 3. Find the general solution for the following ordinary differential equations.

(A) $x\,\dfrac{dy}{dx} = y\,\log|y|$

(B) $xy\,\dfrac{dy}{dx} = 2y^2 + x^2.$

Problem 4. Find the general solution of the ordinary differential equation

$$\frac{dy}{dx} - (\tan x)y = \sec x.$$

Problem 5. Solve the *exact* ordinary differential equation

$$(2xy^3 - y^2 - 2)\,dx + (3x^2y^2 - 2xy + 3)\,dy = 0.$$

Problem 6. For what value of λ is $y = x^\lambda \log(x)$ a solution of the ordinary differential equation $x^2y'' - 5xy' + 9y = 0$?

Problem 7. Solve the ordinary differential equation

$$2xy\frac{dy}{dx} = y^2 + 1.$$

Problem 8. Solve the initial value problem

$$\frac{dy}{dx} = \frac{x^3 + y^3}{xy^2}, \qquad y(1) = 2.$$

Problem 9. Solve the initial value problem

$$\frac{dy}{dx} + y\cos(x) = \cos(x), \qquad y(0) = 3.$$

Problem 10. Make the substitution $z = x + y$ to solve the ordinary differential equation

$$\frac{dy}{dx} = \big(x + y + 2\big)\big(x + y\big).$$

Problem 11. Solve the initial value problem

$$\frac{dy}{dx} = 2 - (2x - y)^2, \qquad y(0) = 1.$$

Hint: The substitution $z = 2x - y$ is illuminating.

Problem 12. Solve the initial value problem

$$(4x + 4y + 3)dx + (4x - 6y - 2)dy = 0, \qquad y(2) = 1.$$

Problem 13. For what values of m and n is $x^m y^n$ an integrating factor for the differential equation

$$(3y + 12xy^2)dx + (4x + 15x^2y)dy = 0.$$

Problem 14. For what values of p and q is $x^p y^q$ an integrating factor for the ordinary differential equation

$$(6y^2 + 3y - 4xy)\,dx + (-3x^2 + 3x + 8xy)\,dy = 0?$$

Problem 15. Find an integrating factor which depends only on y and then solve the differential equation

$$(2y + y^2 - 6xy)\,dx \;+\; (4x + 3xy - 6x^2)\,dy = 0.$$

Problem 16. A 200 L tank initially contains 100 L of brine of concentration 3 g/L salt (i.e., 3 grams salt per liter water). Brine of concentration 5 g/L salt runs into the tank at 8 L/min. The well–mixed solution is drawn off at the rate 6 L/min. Find the concentration of salt in the solution in the tank at the moment that the tank begins to overflow.

Problem 17. A 100 gal tank initially contains 20 gal of brine of concentration 0.24 oz/gal salt. Brine of concentration 0.18 oz/gal flows into the tank at 3 gal/min and the well-mixed solution is drawn off at the rate of 1 gal/min. Find the amount of salt in the tank at the very moment that it begins to overflow.

Problem 18. A brine solution consisting of 0.06 oz/gal salt dissolved in water flows into a large tank at the rate 3.0 gal/min. The solution inside the tank is kept well-mixed and flows out of the tank at the rate 2.0 gal/min. If the tank initially contains 50.0 gal of brine of concentration 0.03 oz/gal determine the amount of salt in the tank after t minutes. When will the concentration of salt in the tank reach 0.05 oz/gal? Assume the tank is so large that it does not overflow.

Problem 19. Find the family of orthogonal trajectories to the one-parameter family of cubics

$$y = \alpha x^3, \qquad \alpha = \text{ arbitrary constant.}$$

Problem 20. Find the family of orthogonal trajectories to the one-parameter family of hyperbolas given by $2y^2 - x^2 = \alpha$.

This collection is not a sample quiz – it's too long. It's just a collection of problems from previous quizzes.

2 Sample Problem Set 2

Problem 21. Find the inverse LAPLACE transform

$$\mathcal{L}^{-1}\left\{\frac{s^2+1}{(s^2-1)(s+1)}\right\}.$$

Problem 22. Consider the initial value problem

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = 4\mathrm{e}^{-t}, \quad y(0) = 2, \quad y'(0) = 12.$$

Find the LAPLACE transform of the solution to this initial value problem.

Problem 23. If

$$\mathcal{L}\{f(t)\} = \frac{s^3}{s^4 - s + 2}$$

compute the LAPLACE transform

$$\mathcal{L}\left\{\mathrm{e}^{-2t} f(t)\right\}.$$

Problem 24. If

$$f(t) = \begin{cases} 3t, & 0 \le t \le 2 \\ 6, & 2 \le t. \end{cases}$$

compute the LAPLACE transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty \mathrm{e}^{-st} f(t)\, dt.$$

Problem 25. Compute and simplify $\mathcal{L}\left\{\mathrm{e}^t \cos t \sin t\right\}$.

Problem 26. Compute and simplify

$$\mathcal{L}^{-1}\left\{\frac{s-1}{s^2}\right\}.$$

Problem 27. Compute and simplify

$$\mathcal{L}^{-1}\left\{\frac{s^2 + s + 6}{(s+1)^2(s-1)}\right\}.$$

Problem 28. Compute and simplify

$$\mathcal{L}^{-1}\left\{\frac{4(s+1)}{s\,(s^2+4)}\right\}.$$

Problem 29. If

$$y(t) + \int_0^t y(r)\, dr = 1$$

use the LAPLACE transform to find $y(t)$.

3 Test 1

Problem 30. Find the general solution of the ordinary differential equations

$$\text{(A)} \quad xy\frac{dy}{dx} = x^2 + 1 \qquad \text{(B)} \quad xy\frac{dy}{dx} = x^2 + y^2.$$

Problem 31. Solve the initial value problem

$$\left(1 + x^2\right)\frac{dy}{dx} + \left(3x + \frac{1}{x}\right)y = 6x + 2, \quad y(1) = 2.$$

Problem 32. Find the orthogonal trajectories for the family of ellipses

$$2y^2 + x^2 = \alpha \quad (\alpha \text{ is an arbitrary parameter}).$$

Problem 33. A large tank contains 80 gallons of brine of concentration 1.621 oz/gal salt. Brine of concentration 2.121 oz/gal salt flows into the tank at 3 gal/min. The well-mixed solution is drawn off at the rate of 4 gal/min. When will the brine in the tank reach a *concentration* of 2.013 oz/gal salt?

Problem 34. Find the general solution of the ordinary differential equation

$$\left(1 + \log(xy)\right)\,dx + \left(1 + \frac{x}{y}\right)\,dy = 0.$$

Problem 35. A thermometer initially reading 62° F is placed in a well insulated cup of very hot coffee. After 2 seconds the thermometer reads 167° F. After an additional 1 second it reads 179° F. If A denotes the temperature of the coffee, T denotes the temperature reading of the thermometer and t denotes time in seconds then according to Newton

$$\frac{dT}{dt} = -k(T - A)$$

where k is a constant. We regard the temperature A of the coffee also as constant. Find the temperature of the coffee.

Problem 36. Relax.

4 Test 2

Problem 37. Find the general solution (*in real form*) for each of the following ordinary differential equations.

(A) $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 4y = 0$ **(B)** $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 3y = 0$ **(C)** $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 5y = 0$

(D) $x^2\dfrac{d^2y}{dx^2} + 5x\dfrac{dy}{dx} + 4y = 0$ **(E)** $x^2\dfrac{d^2y}{dx^2} + 5x\dfrac{dy}{dx} + 3y = 0$ **(F)** $x^2\dfrac{d^2y}{dx^2} + 5x\dfrac{dy}{dx} + 5y = 0$

(G) $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} = 0$ **(H)** $\dfrac{d^2y}{dx^2} + 4y = 0$ **(I)** $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + 4y = 0$

Problem 38. The ordinary differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^{1/2}, \quad x > 0$$

has a particular solution of the form $Ax^{1/2}$ where A is a constant. **(A)** Find the constant A. **(B)** Does the ordinary differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^{-1}, \quad x > 0$$

have a solution of the form Ax^{-1}? Explain your answer.

Problem 39. Find a particular solution of the ordinary differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = x\left(\mathrm{e}^{2x} + \mathrm{e}^{-2x}\right).$$

Problem 40. Consider the ordinary differential equation

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \left(x^2+2\right)y = x^3\sec x, \quad x \geq 0.$$

Given that the complementary solution is

$$c_1 x\cos x + c_2 x\sin x$$

use variation of parameters to find the general solution.

5 Test 3

(Table of LAPLACE Transforms - omitted from this archive.)

Problem 41. Use the table to compute the LAPLACE transforms

$$\textbf{(A)} \quad \mathcal{L}\left\{t^2\mathrm{e}^{3t}\right\} \qquad \textbf{(B)} \quad \mathcal{L}\left\{\mathrm{e}^{2(t-1)}\right\} \qquad \textbf{(C)} \quad \mathcal{L}\left\{(t+1)^3\right\}.$$

Problem 42. Compute the inverse LAPLACE transform

$$\mathcal{L}^{-1}\left\{\frac{2s^2+7s+3}{(s-1)(s+1)(s+2)}\right\}.$$

Problem 43. Compute the inverse LAPLACE transform

$$\mathcal{L}^{-1}\left\{\frac{s^2-4}{(s+1)(s^2+4)}\right\}.$$

Problem 44. Compute the inverse LAPLACE transform

$$\mathcal{L}^{-1}\left\{ \mathrm{e}^{-\pi s/2}\, \frac{s-2}{s^2-4s+13} \right\}.$$

Problem 45. Find the LAPLACE transform of the solution to the initial value problem

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 3y \;=\; \begin{cases} 2 & \text{if } 0 < t < 1 \\ t & \text{if } 1 < t < 4 \\ -1 & \text{if } 4 < t \end{cases}$$
$$y(0) = 1, \quad y'(0) = -2.$$

(Do not solve the differential equation).

6 Contact Information

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