This file contains a few routine sample problems. Be sure you also look at the assignments (model problems). Problems based on the model problems will make up about 3/4'ths of the midterm test. There are no problems involving general integrating factors in this set. Check some of the old tests or problem sets for examples. These problems were thrown together rather quickly. Expect some errors!

Problem 1. Solve the following ordinary differential equations.

(A). $$\frac{dy}{dt} = \frac{\cos(t)}{y+2}$$

(B). $$\frac{dy}{dt} = \frac{y^2+1}{t+2}$$

(C). $$\frac{dy}{dt} = \frac{t^3 y + 2y}{y^4 + y^3 + 1}$$

(D). $$\frac{dy}{dt} + \mathrm{e}^t y = \mathrm{e}^t y^2$$

(E). $$\frac{dy}{dt} = \frac{1+y^2}{\sqrt{1+t^2}}$$

(F). $$\frac{dy}{dt} = \frac{1+t^2}{\sqrt{1+y^2}}$$

(G). $$\cos(y)\frac{dy}{dt} = \sin(t)$$

(H). $$\frac{dy}{dt} = \frac{1-y}{1-t}$$

(I). $$\frac{dy}{dt} = 2y(y-1)$$

(J). $$\frac{dy}{dt} = y(y^2-1)$$

(K). $$\frac{dy}{dt} = y^2(y^2+1)$$

(L). $$t^2\,\frac{dy}{dt} = y^2 - t\,y + t^2$$

(M). $$\frac{dy}{dt} = \frac{2t+3y}{t-2y}$$

(N). $$\left(t^3 - y^3\right)\frac{dy}{dt} = t^2 y$$

(O). $$\frac{dy}{dt} = \sqrt{t+y} + 1, \qquad \text{substitute } w = t + y$$

(P). $$(t^2+4y)\frac{dy}{dt} = 1 - 2ty$$

(Q). $$(\sin(t) + t^2\mathrm{e}^y - 1)\frac{dy}{dt} + y\cos(t) + 2t\mathrm{e}^y + t = 0$$

*Bent Petersen File ref: 256w2001˙samples.tex

Problem 2. Find the general solution of the following BERNOULLI ordinary differential equations. Use the recommended substitution (indicated)

(A). $\dfrac{dy}{dt} + t^{-1}y = y^{-1}, \qquad w = y^2$

(B). $t^2\dfrac{dy}{dt} - ty = y^2, \qquad w = y^{-1}$

(C). $\dfrac{dy}{dt} = y - y^4, \qquad w = y^{-3}$

Problem 3. For each of the following ordinary differential equations find the general solution.

(A). $\dfrac{d^2y}{dt^2} + 6\dfrac{dy}{dt} - 7y = 0$

(B). $\dfrac{d^5y}{dt^5} - \dfrac{d^4y}{dt^4} - 11\dfrac{d^3y}{dt^3} + 29\dfrac{d^2y}{dt^2} - 26\dfrac{dy}{dt} + 8y = 0$

(C). $2\dfrac{d^2y}{dt^2} - \dfrac{1}{2}y = 0$

(D). $\dfrac{d^2y}{dt^2} - y = 0$

(E). $\dfrac{d^2y}{dt^2} - \dfrac{dy}{dt} = 0$

(F). $\dfrac{d^3y}{dt^3} + 2\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} + 8y = 0$

(G). $\dfrac{d^4y}{dt^4} - y = 0$

(H). $\dfrac{d^4y}{dt^4} + 2\dfrac{d^2y}{dt^2} + y = 0$

(I). $\dfrac{d^4y}{dt^4} + y = 0$

(J). $\dfrac{d^2y}{dt^2} - 6\dfrac{dy}{dt} + 25y = 0$

For part (I) note that $\lambda^4 + 1 = (\lambda^2+1)^2 - \left(\sqrt{2}\lambda\right)^2$.

Problem 4. For each of the following initial value problems find the solution.

(A). $\dfrac{d^2y}{dt^2} + 6\dfrac{dy}{dt} - 7y = 0, \qquad y(0) = 0, y'(0) = -2$

(B). $\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} + 5y = 0, \qquad y(0) = -1, y'(0) = 1$

(C). $\dfrac{d^4y}{dt^4} - 16y = 0, \qquad y(0) = 1, y'(0) = 0, y''(0) = 0, y'''(0) = 1$

(D). $\cos(2t)\dfrac{d^2y}{dt^2} + \dfrac{\arctan(t)}{t^2+4}\dfrac{dy}{dt} + \sin(2t)y = 0, \qquad y(0) = 0, y'(0) = 0$