Linear ODE with Constant Coefficients
Method of Undetermined Coefficients

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Maple 6

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For each of the following Cauchy initial value problems do the following: (1) Find the complementary solution, that is, a general solution of the associated homogeneous equation; (2) Use the method of undetermined coefficients to find a particular solution of the inhomogeneous equation; (3) Combine the results of the first two parts to find a general solution of the inhomogeneous equation; (4) Determine the values of the parameters (arbitrary constants) in the general solution found in the previous step so as to satisfy the initial values condition.

Does your solution agree with Maple's solution?

Problem 1

```
> ode01:=diff(y(x),x,x)+3*diff(y(x),x)+2*y(x)=x^2+1;
```

$$ode01 := \left(\frac{\partial^2}{\partial x^2} y(x)\right) + 3\left(\frac{\partial}{\partial x} y(x)\right) + 2\,y(x) = x^2 + 1$$

```
> init01:=y(0)=3,D(y)(0)=-1;
```

$$init01 := y(0) = 3,\ D(y)(0) = -1$$

```
> dsolve({ode01,init01},y(x));
```

$$y(x) = \frac{1}{2}x^2 - \frac{3}{2}x + \frac{9}{4} - \frac{5}{4}\mathbf{e}^{(-2x)} + 2\,\mathbf{e}^{(-x)}$$

Problem 2

```
> ode02:=diff(y(x),x,x)-diff(y(x),x)=1+exp(x);
```

$$ode02 := \left(\frac{\partial^2}{\partial x^2} y(x)\right) - \left(\frac{\partial}{\partial x} y(x)\right) = 1 + \mathbf{e}^{x}$$

```
> init02:=y(0)=1,D(y)(0)=2;
```

$$init02 := y(0) = 1, D(y)(0) = 2$$

```
> dsolve({ode02,init02},y(x));
```

$$y(x) = -x + e^{x} x + 2 e^{x} - 1$$

Problem 3

```
> ode03:=diff(y(x),x,x)+4*y(x)=x*cos(2*x)+x*cos(x);
```

$$ode03 := \left(\frac{\partial^2}{\partial x^2} y(x)\right) + 4\, y(x) = x \cos(2\, x) + x \cos(x)$$

```
> init03:=y(0)=A,D(y)(0)=B;
```

$$init03 := y(0) = A,\ D(y)(0) = B$$

```
> dsolve({ode03,init03},y(x));
```

$$y(x) = \left(-\frac{169}{576} + \frac{1}{2} B\right) \sin(2\, x) + A \cos(2\, x) + \left(-\frac{1}{64} + \frac{1}{8} x^2\right) \sin(2\, x) + \frac{1}{3} x \cos(x) + \frac{1}{16} x \cos(2\, x) + \frac{2}{9} \sin(x)$$

Problem 4

```
> ode04:=diff(y(x),x,x)+2*diff(y(x),x)+10*y(x)=exp(-x)+exp(-x)*sin(3
  *x);
```

$$ode04 := \left(\frac{\partial^2}{\partial x^2} y(x)\right) + 2\left(\frac{\partial}{\partial x} y(x)\right) + 10\, y(x) = e^{(-x)} + e^{(-x)} \sin(3\, x)$$

```
> init04:=y(0)=1,D(y)(0)=-2;
```

$$init04 := y(0) = 1,\ D(y)(0) = -2$$

```
> dsolve({ode04,init04},y(x));
```

$$y(x) = \frac{1}{9} e^{(-x)} - \frac{5}{18} e^{(-x)} \sin(3\, x) - \frac{1}{6} e^{(-x)} \cos(3\, x)\, x + \frac{8}{9} e^{(-x)} \cos(3\, x)$$

Problem 5

```
> ode05:=diff(y(x),x,x)-4*y(x)=exp(2*x)-exp(-2*x)+exp(x)-exp(-x);
```

$$ode05 := \left(\frac{\partial^2}{\partial x^2} y(x)\right) - 4\, y(x) = e^{(2\, x)} - e^{(-2\, x)} + e^{x} - e^{(-x)}$$

```
> init05:=y(0)=0,D(y)(0)=0;
```

$$init05 := y(0) = 0,\ D(y)(0) = 0$$

```
> dsolve({ode05,init05},y(x)): simplify(%);
```

$$y(x) = \frac{1}{24} e^{(2\, x)} + \frac{1}{4} e^{(-2\, x)} x - \frac{1}{3} e^{x} + \frac{1}{3} e^{(-x)} + \frac{1}{4} x\, e^{(2\, x)} - \frac{1}{24} e^{(-2\, x)}$$

Problem 6

```
> ode06:=diff(y(x),x$3)+3*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=x+x*exp
  (-x);
```

$$ode06 := \left(\frac{\partial^3}{\partial x^3} y(x)\right) + 3\left(\frac{\partial^2}{\partial x^2} y(x)\right) + 3\left(\frac{\partial}{\partial x} y(x)\right) + y(x) = x + x\,\mathbf{e}^{(-x)}$$

```
> init06:=y(0)=2,D(y)(0)=-1,(D@@2)(y)(0)=2;
```

$$init06 := y(0) = 2,\ D(y)(0) = -1,\ (D^{(2)})(y)(0) = 2$$

```
> dsolve({ode06,init06},y(x));
```

$$y(x) = x - 3 + \frac{1}{24}\mathbf{e}^{(-x)} x^4 + 5\,\mathbf{e}^{(-x)} + 3\,x\,\mathbf{e}^{(-x)} + \frac{3}{2}\mathbf{e}^{(-x)} x^2$$

Problem 7

```
> ode07:=diff(y(x),x$4)-y(x)=x+exp(x)-exp(-x)+cos(x)-sin(x);
```

$$ode07 := \left(\frac{\partial^4}{\partial x^4} y(x)\right) - y(x) = x + \mathbf{e}^{x} - \mathbf{e}^{(-x)} + \cos(x) - \sin(x)$$

```
> init07:=y(0)=1,D(y)(0)=-3,(D@@2)(y)(0)=2,(D@@3)(y)(0)=-1;
```

$$init07 := y(0) = 1,\ D(y)(0) = -3,\ (D^{(2)})(y)(0) = 2,\ (D^{(3)})(y)(0) = -1$$

```
> dsolve({ode07,init07},y(x)): simplify(%);
```

$$y(x) = \frac{1}{2}\sin(x) - \frac{1}{2}\mathbf{e}^{x} - \frac{3}{4}\cos(x) + \frac{9}{4}\mathbf{e}^{(-x)} - \frac{1}{4}\sin(x)\,x - \frac{1}{4}x\cos(x) + \frac{1}{4}x\,\mathbf{e}^{(-x)} - x + \frac{1}{4}x\,\mathbf{e}^{x}$$

Problem 8

```
> ode08:=diff(y(x),x$3)-6*diff(y(x),x$2)-9*diff(y(x),x)+14*y(x)=exp(
  x)+exp(-x);
```

$$ode08 := \left(\frac{\partial^3}{\partial x^3} y(x)\right) - 6\left(\frac{\partial^2}{\partial x^2} y(x)\right) - 9\left(\frac{\partial}{\partial x} y(x)\right) + 14\,y(x) = \mathbf{e}^{x} + \mathbf{e}^{(-x)}$$

```
> init08:=y(log(2))=0,D(y)(log(2))=0,(D@@2)(y)(log(2))=0;
```

$$init08 := y(\ln(2)) = 0,\ D(y)(\ln(2)) = 0,\ (D^{(2)})(y)(\ln(2)) = 0$$

```
> dsolve({ode08,init08},y(x));
```

$$y(x) = \left(-\frac{1}{18}\mathbf{e}^{(9x)} x + \frac{1}{16}\mathbf{e}^{(7x)} + \frac{1}{108}\mathbf{e}^{(9x)}\right)\mathbf{e}^{(-8x)} + \left(\frac{1}{18}\ln(2) - \frac{1}{144}\right)\mathbf{e}^{x} - \frac{14}{81}\mathbf{e}^{(-2x)} + \frac{19}{331776}\mathbf{e}^{(7x)}$$

Problem 9

```
> ode09:=diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=x^2*exp(x);
```

$$ode09 := \left(\frac{\partial^2}{\partial x^2} y(x)\right) - 5\left(\frac{\partial}{\partial x} y(x)\right) + 6\, y(x) = x^2\, \mathbf{e}^x$$

```
> init09:=y(0)=2,D(y)(0)=3;
```

$$init09 := y(0) = 2,\ D(y)(0) = 3$$

```
> dsolve({ode09,init09},y(x));
```

$$y(x) = \frac{7}{4}\mathbf{e}^x + \frac{3}{2}x\,\mathbf{e}^x + \frac{1}{2}x^2\,\mathbf{e}^x + \mathbf{e}^{(2x)} - \frac{3}{4}\mathbf{e}^{(3x)}$$

Problem 10

```
> ode10:=diff(y(x),x$2)+y(x)=sin(x)+cos(x);
```

$$ode10 := \left(\frac{\partial^2}{\partial x^2} y(x)\right) + y(x) = \sin(x) + \cos(x)$$

```
> init10:=y(0)=A,D(y)(0)=B;
```

$$init10 := y(0) = A,\ D(y)(0) = B$$

```
> dsolve({ode10,init10},y(x)): simplify(%);
```

$$y(x) = \frac{1}{2}\sin(x) + \sin(x)\,B + \cos(x)\,A + \frac{1}{2}\sin(x)\,x - \frac{1}{2}x\cos(x)$$

```
>
```