

You may use a notesheet, prepared in advance, and no larger than  $8.5 \times 11$  inches in size. You are expected to have a scientific calculator, and you may use it. Please note  $\log(x)$  means the natural logarithm of  $x$  on this test.

This test is multiple-choice. Work carefully. Try to avoid errors and try to avoid being misled by the offered answers.

There are 8 problems for a total of 160 points.

**Problem 1.** (20 points if correct, 0 points if wrong). Find the general solution of the differential equation

$$x \frac{dy}{dx} = (1 + x^2)(1 + y^2).$$

- A.)**  $y = \tan(\arctan(x) + C)$       **B.)**  $y = \tan\left(x + \frac{x^3}{3} + C\right)$   
**C.)**  $y = \tan\left(\frac{x^2}{2} + \log(x) + C\right)$       **D.)**  $y = \log\left(\frac{x^2}{2} + \tan(x) + C\right)$       **E.)** None of the foregoing.

←Letter corresponding to your answer to problem 1.

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**Problem 2.** (20 points if correct, 0 points if wrong). Solve the initial value problem

$$\frac{dy}{dx} = e^{x-y}, \quad y(0) = \log 2.$$

- A.)**  $y = \log(2 + x)$       **B.)**  $y = \log(1 + e^x)$   
**C.)**  $y = \log(2) - 1 + e^x$       **D.)**  $y = \log(2) - e + e^{1+x}$       **E.)** None of the foregoing.

←Letter corresponding to your answer to problem 2.

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**Problem 3.** (20 points if correct, 0 points if wrong). Solve the differential equation

$$x \frac{dy}{dx} + y = 2x.$$

- A.)**  $y = x^2 + C$       **B.)**  $y = x + C$   
**C.)**  $y = 2 + C/x$       **D.)**  $y = x + C/x$       **E.)** None of the foregoing.

←Letter corresponding to your answer to problem 3.

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**Problem 4.** (20 points if correct, 0 points if wrong). Solve the exact ordinary differential equation

$$2x + y + (x + 6y) \frac{dy}{dx} = 0$$

- A.)**  $x^2 + xy + y^2 = C$       **B.)**  $x^2 + xy + 3y^2 = C$   
**C.)**  $x^2 + xy + 6y^2 = C$       **D.)**  $x^2 + 4xy + 3y^2 = C$       **E.)** None of the foregoing.

←Letter corresponding to your answer to problem 4.

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**Problem 5.** (20 points if correct, 0 points if wrong). The ordinary differential equation

$$y(x + y + 1) + (x + 2y)\frac{dy}{dx} = 0$$

has an integrating factor depending only on  $x$ . Find such an integrating factor.

- A.)**  $x$       **B.)**  $x^2$   
**C.)**  $e^x$     **D.)**  $\log(x)$     **E.)** None of the foregoing.

←Letter corresponding to your answer to problem 5.

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**Problem 6.** (20 points if correct, 0 points if wrong). If we substitute  $y = xv$  in the differential equation

$$\frac{dy}{dx} = \frac{xy + y^2}{x^2}$$

we obtain

- A.)**  $x \frac{dv}{dx} = v^2$       **B.)**  $x \frac{dv}{dx} = v^2 + v$   
**C.)**  $x \frac{dv}{dx} = v^2 + 2v$     **D.)**  $\frac{dv}{dx} = v^2$       **E.)** None of the foregoing.

←Letter corresponding to your answer to problem 6.

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**Problem 7.** (20 points if correct, 0 points if wrong). A very large tank contains 40 L brine of concentration 2 g/L salt. Brine of concentration 1 g/L salt flows into the tank at 3 L/min and the well-mixed solution is pumped out at 2 L/min. Assuming that the tank does not overflow what is the concentration of salt in the brine in the tank after 10 min? (Choose the closest value.)

- A.)** 1.500 g/L    **B.)** 1.512 g/L  
**C.)** 1.643 g/L    **D.)** 1.883 g/L    **E.)** 1.914 g/L

←Letter corresponding to your answer to problem 7.

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**Problem 8.** (20 points if correct, 0 points if wrong). Given the initial value problem

$$\frac{dy}{dx} = x^2 + y^2, \quad y(0) = 1$$

estimate  $y(0.6)$  by using EULER's method with step size  $h = 0.2$ . Choose the closest number from the list below.

- A.)** 1.97560    **B.)** 2.22237  
**C.)** 2.31246    **D.)** 2.58132    **E.)** 2.64399

←Letter corresponding to your answer to problem 8.

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