

You may use a notesheet, prepared in advance, and no larger than 8.5×11 inches in size. You are expected to have a scientific calculator, and you may use it. Please note $\log(x)$ means the natural logarithm of x on this test.

This test is multiple-choice. Work carefully. Try to avoid errors and try to avoid being misled by the offered answers.

There are 10 problems for a total of 160 points.

Problem 1. (16 points if correct, 0 points if wrong). Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 13y = 0$$

- A.) $y = c_1e^{2x} + c_2xe^{2x}$ B.) $y = c_1 + c_2e^{4x}$
C.) $y = c_1e^{-x} + c_2e^{5x}$ D.) $y = c_1e^{2x} \cos(3x) + c_2e^{2x} \sin(3x)$ E.) None of the foregoing.

←Letter corresponding to your answer to problem 1.

Problem 2. (16 points if correct, 0 points if wrong). Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 4y = 0$$

- A.) $y = c_1e^{2x} + c_2xe^{2x}$ B.) $y = c_1 + c_2e^{4x}$
C.) $y = c_1e^{-x} + c_2e^{5x}$ D.) $y = c_1e^{2x} \cos(3x) + c_2e^{2x} \sin(3x)$ E.) None of the foregoing.

←Letter corresponding to your answer to problem 2.

Problem 3. (16 points if correct, 0 points if wrong). Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} - 5y = 0$$

- A.) $y = c_1e^{2x} + c_2xe^{2x}$ B.) $y = c_1 + c_2e^{4x}$
C.) $y = c_1e^{-x} + c_2e^{5x}$ D.) $y = c_1e^{2x} \cos(3x) + c_2e^{2x} \sin(3x)$ E.) None of the foregoing.

←Letter corresponding to your answer to problem 3.

Problem 4. (16 points if correct, 0 points if wrong). Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} = 0$$

- A.) $y = c_1e^{2x} + c_2xe^{2x}$ B.) $y = c_1 + c_2e^{4x}$
C.) $y = c_1e^{-x} + c_2e^{5x}$ D.) $y = c_1e^{2x} \cos(3x) + c_2e^{2x} \sin(3x)$ E.) None of the foregoing.

←Letter corresponding to your answer to problem 4.

Problem 5. (16 points if correct, 0 points if wrong). The differential equation

$$\frac{d^5y}{dx^5} + \frac{d^4y}{dx^4} - 2\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = xe^x + xe^{-x}$$

has characteristic roots 1, 1, -1, -1, -1 (repeated according to multiplicity). Find the form of a particular solution as specified by the method of undetermined coefficients.

- A.)** $(Ax + B)e^x + (Cx + D)e^{-x}$ **B.)** $x^2(Ax + B)e^x + x^2(Cx + D)e^{-x}$
C.) $Ax^3e^x + Bx^4e^{-x}$ **D.)** $Axe^x + Bxe^{-x}$ **E.)** None of the foregoing.

←Letter corresponding to your answer to problem 5.

Problem 6. (16 points if correct, 0 points if wrong). The differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = xe^{-x}$$

has characteristic roots $-1 + 4i$, $-1 - 4i$. Find the form of a particular solution as specified by the method of undetermined coefficients.

- A.)** $x(Ax + B)e^{-x}$ **B.)** $(Ax + B)e^{-x}$
C.) Axe^{-x} **D.)** Ax^2e^{-x} **E.)** None of the foregoing.

←Letter corresponding to your answer to problem 6.

Problem 7. (16 points if correct, 0 points if wrong). The differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = \cos(x)e^{-2x}$$

has characteristic roots -2 , -2 (repeated according to multiplicity). Find the form of a particular solution as specified by the method of undetermined coefficients.

- A.)** $A\cos(x)e^{-2x} + B\sin(x)e^{-2x}$ **B.)** $Ax^2\cos(x)e^{-2x} + Bx^2\sin(x)e^{-2x}$
C.) $A\cos(x)e^{-2x}$ **D.)** $Ax^2\cos(x)e^{-2x}$ **E.)** None of the foregoing.

←Letter corresponding to your answer to problem 7.

Problem 8. (16 points if correct, 0 points if wrong). The differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = \cos(x)e^{-2x}$$

has characteristic roots $-2 + i$, $-2 - i$. Find the form of a particular solution as specified by the method of undetermined coefficients.

- A.)** $A\cos(x)e^{-2x} + B\sin(x)e^{-2x}$ **B.)** $Ax^2\cos(x)e^{-2x} + Bx^2\sin(x)e^{-2x}$
C.) $A\cos(x)e^{-2x}$ **D.)** $Ax^2\cos(x)e^{-2x}$ **E.)** None of the foregoing.

←Letter corresponding to your answer to problem 8.

Problem 9. (16 points if correct, 0 points if wrong). Find the general solution of the differential equation

$$y \frac{d^2 y}{dx^2} - \left(\frac{dy}{dx} \right)^2 = 0$$

- A.)** $y = C_1 + C_2 \exp(x)$ **B.)** $y = C_1 + \exp(C_2 x)$
C.) $y = C_1 \exp(C_2 x)$ **D.)** $y = C_1 + C_2 x$ **E.)** None of the foregoing.

← Letter corresponding to your answer to problem 9.

Problem 10. (16 points if correct, 0 points if wrong). The differential equation

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = 4x^{-1}$$

has the complementary solution $C_1 x + \frac{C_2}{x}$. Use variation of parameters to find a particular solution

- A.)** $x + x^{-1}$ **B.)** $x^{-1} \log(x)$
C.) $y = \frac{1 + 2 \log(x)}{x}$ **D.)** $y = 1 + 2 \log(x)$ **E.)** None of the foregoing.

← Letter corresponding to your answer to problem 10.
