

**Instructions:**  $\implies$ 

If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to write your name in the space above.

- You may use one  $8.5 \times 11$  inch note sheet prepared in advance. You may write on both sides of your note sheet.
- Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes.
- You may not use any books, notebooks, additional note sheets nor note cards.
- You are expected to have a simple scientific calculator available for use on this test. Calculators and other equipment may not be shared.
- For multiple-choice problems place the letter corresponding to your answer in the box provided.
- Note that  $\log(x)$  means the natural logarithm of  $x$ .

**Problem 1.** (10 points if correct, -1 points if wrong). Solve the initial value problem

$$\frac{dy}{dt} + \frac{1}{t}y = t^2, \quad y(1) = 1.$$

Then choose the number in the list below closest to  $y(2)$ .

- A.) 1.000    B.) 3.000  
C.) 2.375    D.) 3.125    E.) 3.250

$\leftarrow$  Letter corresponding to your answer to problem 1.

**Problem 2.** (10 points if correct, -1 points if wrong). Solve the initial value problem

$$\frac{dy}{dt} + y^2 = -y, \quad y(0) = 3.$$

- A.)  $2e^{-t} + 1$     B.)  $\frac{2}{3e^t - 2}$   
C.)  $\frac{3}{4e^t - 3}$     D.)  $\frac{1}{2e^t - 1}$     E.) None of the above.

$\leftarrow$  Letter corresponding to your answer to problem 2.

**Problem 3.** (10 points if correct, -1 points if wrong). Solve the exact ordinary differential equation

$$-4e^{-2x} \cos(3y) + 3 + (-6e^{-2x} \sin(3y) - 4) \frac{dy}{dx} = 0.$$

- A.) Not exact!    B.)  $2e^{-2x} \cos(3y) + 3x - 4y = c$   
C.)  $4e^{-2x} \cos(3y) + 3y - 4x = c$     D.)  $4e^{-2x} \cos(3y) + 12xy = c$     E.) None of the above.

$\leftarrow$  Letter corresponding to your answer to problem 3.

**Problem 4.** (10 points if correct, -1 points if wrong). The ordinary differential equation

$$4y + 15xy^2 - 49x^3y + (3x + 12yx^2 - 21x^4) \frac{dy}{dx} = 0$$

has an integrating factor of the form  $\mu = x^m y^n$ . Find  $m$  and  $n$ .

- A.)**  $m = 3, n = 4$     **B.)**  $m = 2, n = -3$   
**C.)**  $m = 3, n = 2$     **D.)**  $m = 2, n = 3$     **E.)** None of the above.

←Letter corresponding to your answer to problem 4.

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**Problem 5.** (10 points if correct, -1 points if wrong). A 100 gallon tank initially contains 50 gallons of brine of concentration 1.5 oz salt per gallon. Brine of concentration 3 oz salt per gallon flows into the tank at 4 gallons per minute. The well-mixed solution is pumped out at 2 gallons per minute. Find the concentration of salt in the brine in the tank at the very moment of overflow. Select the closest number from the list below.

- A.)** 2.625    **B.)** 3.175  
**C.)** 3.312    **D.)** 2.985    **E.)** 2.758

←Letter corresponding to your answer to problem 5.

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**Problem 6.** (10 points if correct, -1 points if wrong). A cup of coffee initially at temperature  $T_0 = 172^\circ\text{F}$  is brought into a room at temperature  $A = 72^\circ\text{F}$ . The heat capacity of the room (compared to the coffee) is so large that we may regard  $A$  as being constant. After 6 minutes the temperature of the coffee is  $140^\circ\text{F}$ . What temperature will the coffee be an additional 4 minutes later? Choose the closest value from the list below.

- A.)**  $120^\circ\text{F}$     **B.)**  $125^\circ\text{F}$   
**C.)**  $130^\circ\text{F}$     **D.)**  $135^\circ\text{F}$     **E.)**  $140^\circ\text{F}$

←Letter corresponding to your answer to problem 6.

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**Problem 7.** (10 points if correct, -1 points if wrong). Find the general solution of the ordinary differential equation

$$\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 12y = 0.$$

- A.)**  $y = c_1e^{-3t} + c_2e^{-2t} + c_3e^{2t}$     **B.)**  $y = c_1e^{3t} + c_2e^{2t} + c_3te^{2t}$   
**C.)**  $y = c_1e^{3t} + c_2e^{2t} + c_3e^{-2t}$     **D.)**  $y = c_1e^t + c_2te^t + c_3t^2e^t$     **E.)** None of the above.

←Letter corresponding to your answer to problem 7.

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**Problem 8.** (10 points if correct, -1 points if wrong). Solve the ordinary differential equation

$$\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2} - 8\frac{dy}{dt} + 12y = 0.$$

- A.)**  $y = c_1e^{2t} + c_2te^{2t} + c_3t^2e^{2t}$     **B.)**  $y = c_1e^{3t} + c_2e^{2t} + c_3te^{2t}$   
**C.)**  $y = c_1e^{-3t} + c_2e^{-2t} + c_3te^{-2t}$     **D.)**  $y = c_1e^{-3t} + c_2e^{2t} + c_3te^{2t}$     **E.)** None of the above.

←Letter corresponding to your answer to problem 8.

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This page is for scratch work. Use also the backs of the test pages.

1	2	3	4	5	6	7	8
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8 problems 80 points	TOTAL:
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