Mth 256 Test 2 | Name: ID:

Bent Petersen `256w2004-test-2.tex` March 4, 2004 Time: 50 minutes.

Instructions: $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to write your name in the space above.

- *You may use one* 8.5×11 *inch note sheet prepared in advance. You may write on both sides of your note sheet.*
- *Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes.*
- *You may not use any books, notebooks, additional note sheets nor note cards.*
- *You are expected to have a simple scientific calculator available for use on this test. Calculators and other equipment may not be shared.*
- *For multiple-choice problems place the letter corresponding to your answer in the box provided.*
- *Note that* $\log(x)$ *means the* natural logarithm *of* x.

Problem 1. *(10 points if correct, -1 points if wrong).* The ordinary differential equation

$$x\,\frac{d^2y}{dx^2} - \frac{dy}{dx} + \frac{1}{x}\,y = 2x$$

has a solution of the form $y = Ax^2$ where A is a constant. Find A.

A.) 1 **B.)** 2
C.) 3 **D.)** 4 **E.)** None of the above.

☐ $\leftarrow$*Letter corresponding to your answer to problem 1.*

Problem 2. *(10 points if correct, -1 points if wrong).* Consider the ordinary differential equation

$$x^2\,\frac{d^2y}{dx^2} - 4x\,\frac{dy}{dx} + 6y = x^3.$$

Use the method of variation of parameters to find a particular solution given that the associated homogeneous equation (or complementary equation) has fundamental solution set $\{x^2,\, x^3\}$. *Note the particular solution that you compute may not coincide precisely with the correct expression below. Be flexible.*

A.) $2x^2 - 4x^3 + 3x^5$ **B.)** $\frac{1}{3}\,x^3$
C.) $-x^3 + x^3\log(x)$ **D.)** $-x^3 + x^2\log(x)$ **E.)** None of the above.

☐ $\leftarrow$*Letter corresponding to your answer to problem 2.*

Problem 3. *(5 points if correct, -1 points if wrong).* Let y_1 and y_2 be solutions of $y'' + p(t)y' + q(t)y = 0$ on the interval (a, b), where p and q are continuous on the interval (a, b). Suppose there is a point t_0 in (a, b) such that $y_1'(t_0) = 0$ and $y_2'(t_0) = 0$, Then

A.) If W is the Wronskian of y_1 and y_2 then $W(t) \neq 0$ for each t in (a, b)

B.) y_1 and y_2 are linearly independent on (a, b)

C.) y_1 is a constant multiple of y_2 or y_2 is a constant multiple of y_1

D.) y_1 and y_2 are different colors.

E.) None of the above.

☐ ←*Letter corresponding to your answer to problem 3.*

Problem 4. *(60 points).* For each of the following inhomogeneous linear ordinary differential equations determine precisely the *form* of the particular solution given by the rules of the method of undetermined coefficients (judicious guessing). Select your answer from the list below and place the corresponding letter in the provided box.

	$y'' + 6y' + 13y = x\mathrm{e}^{-3x}$		$y'' + 6y' + 9y = x\mathrm{e}^{-3x}$
	$y'' + y' + 5y = x\,\mathrm{e}^{-x}$		$y'' + 3y' + 2y = \cos(2x)$
	$y'' + 4y = x\,\sin(2x)$		$y''' + 3y'' + 3y' + y = \mathrm{e}^{-x}$

Answers:

A	$(Ax + B)\mathrm{e}^{-3x}$	**B**	$x(Ax + B)\mathrm{e}^{-3x}$
C	$x^2(Ax + B)\mathrm{e}^{-3x}$	**D**	$A\mathrm{e}^{-x}$
E	$Ax\mathrm{e}^{-x}$	**F**	$Ax^2\mathrm{e}^{-x}$
G	$Ax^3\mathrm{e}^{-x}$	**H**	$(Ax + B)\mathrm{e}^{-x}$
I	$x(Ax + B)\mathrm{e}^{-x}$	**J**	$x^2(Ax + B)\mathrm{e}^{-x}$
K	$A\cos(2x) + B\sin(2x)$	**L**	$Ax\cos(2x) + Bx\sin(2x)$
M	$x(Ax + B)\cos(2x) + x(Cx + D)\sin(2x)$	**N**	None of the foregoing

This page is for scratch work. Use also the backs of the test pages.

1 (10)	2 (10)	3 (5)	4 (60)

4 problems 85 points	TOTAL: