

Applied Differential Equations

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The problems on this list may be done by the method of variation of parameters.

For each problem find the general solution by using the method of variation of parameters. In those cases where it is feasible, use also the method of undetermined coefficients.

In some cases below the integration, or even just the algebra, may be challenging. You may grow noticeably older working out the solution. On a test I may just ask you for the *form* of the solution, rather than the actual solution, so you will have time to do more than one problem!

The solutions provided below have not been proof-read. There may be some typos. If your solution appears not to agree with mine, check first to see if it is equivalent. If not, then check your work. If it checks out then you may have found an error in my work. By all means let me know.

Problem 1.

$$\frac{d^2y}{dx^2} + y = \sec(x)$$

Solution 1.

$$y(x) = c_1 \cos(x) + c_2 \sin(x) + x \sin(x) + \cos(x) \log(\cos(x))$$

Problem 2.

$$\frac{d^2y}{dx^2} + y = \tan(x)$$

Solution 2.

$$y(x) = c_1 \cos(x) + c_2 \sin(x) - \cos(x) * \log(\sec(x) + \tan(x))$$

Problem 3.

$$\frac{d^2y}{dx^2} + y = \tan(x) \tan(x)$$

Solution 3.

$$y(x) = c_1 \cos(x) + c_2 \sin(x) + \log(\sec(x)) \sin(x) + x \cos(x)$$

Problem 4.

$$\frac{d^2y}{dx^2} + y = \tan^2(x)$$

Solution 4.

$$y(x) = c_1 \cos(x) + c_2 \sin(x) - 2 + \sin(x) \log(\sec(x) + \tan(x))$$

Problem 5.

$$\frac{d^2y}{dx^2} + y = \sec^2(x)$$

Solution 5.

$$y(x) = c_1 \cos(x) + c_2 \sin(x) - 1 + \sin(x) \log(\sec(x) + \tan(x))$$

Problem 6.

$$\frac{d^2y}{dx^2} + y = \cos^2(x)$$

Solution 6.

$$y(x) = c_1 \cos(x) + c_2 \sin(x) + \frac{1}{2} - \frac{1}{6} \cos(2x)$$

Problem 7.

$$\frac{d^2y}{dx^2} - 4y = e^{2x}$$

Solution 7.

$$y(x) = c_1 e^{2x} + c_2 e^{-2x} + \frac{1}{4} x e^{2x}$$

Problem 8.

$$\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = \frac{e^x}{1 + e^x}$$

Solution 8.

$$y(x) = c_1 e^{-2x} + c_2 e^{-x} + \frac{1}{2} - e^{-2x} \log(1 + e^x) - e^{-x} \log(1 + e^x)$$

Problem 9.

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = e^x \arctan(x)$$

Solution 9.

$$y(x) = c_1 e^x + c_2 x e^x + \frac{e^x}{2} (\arctan(x)x^2 + \arctan(x) - x \log(1 + x^2))$$

Problem 10.

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = e^x \sec(x)$$

Solution 10.

$$y(x) = c_1 e^x \cos(x) + c_2 e^x \sin(x) + e^x (x \sin(x) + \log(\cos(x)) \cos(x))$$

Problem 11.

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 4x \log(x)$$

Hint: The complementary solution is $y(x) = c_1 x + c_2 x \log(x)$.

Solution 11.

$$y(x) = c_1 x + c_2 x \log(x) + \frac{2x}{3} \log^3(x)$$

Problem 12.

$$x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 6y = 4x^3$$

Hint: The complementary solution is $y(x) = c_1 x^2 + c_2 x^3$.

Solution 12.

$$y(x) = c_1 x^2 + c_2 x^3 + 4x^3 (\log(x) - 1)$$

Problem 13.

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + \left(x^2 - \frac{1}{4}\right) y(x) = x^{\frac{3}{2}}$$

Hint: The complementary solution is $y(x) = c_1 \frac{\cos(x)}{\sqrt{x}} + c_2 \frac{\sin(x)}{\sqrt{x}}$.

Solution 13.

$$y(x) = c_1 \frac{\cos(x)}{\sqrt{x}} + c_2 \frac{\sin(x)}{\sqrt{x}} + \frac{1}{\sqrt{x}}$$

Problem 14.

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + \left(x^2 - \frac{1}{4}\right) y(x) = x^{\frac{3}{2}} \sin(x)$$

Hint: The complementary solution is $y(x) = c_1 \frac{\cos(x)}{\sqrt{x}} + c_2 \frac{\sin(x)}{\sqrt{x}}$.

Solution 14.

$$y(x) = c_1 \frac{\cos(x)}{\sqrt{x}} + c_2 \frac{\sin(x)}{\sqrt{x}} + \frac{1}{2} \frac{\sin(x) - x \cos(x)}{\sqrt{x}}$$

Problem 15.

$$\frac{d^3 y}{dx^3} - 3 \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} = e^x$$

Solution 15.

$$y(x) = c_1 e^{2x} + c_2 e^x + c_3 - x e^x$$

Problem 16.

$$\frac{d^4 y}{dx^4} - y = \sin(x)$$

Solution 16.

$$y(x) = c_1 e^x + c_2 e^{-x} + c_3 \cos(x) + c_4 \sin(x) + \frac{1}{4} x \cos(x)$$

Problem 17.

$$\frac{d^4 y}{dx^4} + y = e^{\frac{\sqrt{2}x}{2}} \sin\left(\frac{\sqrt{2}x}{2}\right)$$

Solution 17.

$$\begin{aligned} y(x) = & c_1 e^{\frac{\sqrt{2}x}{2}} \sin\left(\frac{\sqrt{2}x}{2}\right) + c_2 e^{\frac{\sqrt{2}x}{2}} \cos\left(\frac{\sqrt{2}x}{2}\right) + c_3 e^{-\frac{\sqrt{2}x}{2}} \sin\left(\frac{\sqrt{2}x}{2}\right) \\ & + c_4 e^{-\frac{\sqrt{2}x}{2}} \cos\left(\frac{\sqrt{2}x}{2}\right) - \frac{\sqrt{2}x}{8} e^{\frac{\sqrt{2}x}{2}} \cos\left(\frac{\sqrt{2}x}{2}\right) - \frac{\sqrt{2}x}{8} e^{\frac{\sqrt{2}x}{2}} \sin\left(\frac{\sqrt{2}x}{2}\right) \end{aligned}$$

Problem 18.

$$\frac{d^2 y}{dx^2} + y = (x^4 + 1) \cos(2x)$$

Solution 18.

$$y(x) = c_1 \cos(x) + c_2 \sin(x) + \frac{-27x^4 + 468x^2 - 995}{81} \cos(2x) + \frac{144x^3 - 960x}{81} \sin(2x)$$

Problem 19.

$$\frac{d^2y}{dx^2} + y = (x^4 + 1) \cos(x)$$

Solution 19.

$$y(x) = c_1 \cos(x) + c_2 \sin(x) + \frac{5x^4 - 15x^2}{20} \cos(x) + \frac{2x^4 - 10x^2 + 25}{20} x \sin(x)$$

Problem 20.

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = x(e^{3x} + e^{-3x})$$

Solution 20.

$$y(x) = c_1 e^{2x} + c_2 e^{-3x} + \left(\frac{1}{6}x - \frac{7}{36}\right) e^{3x} - \left(\frac{1}{10}x + \frac{1}{25}\right) x e^{-3x}$$

Problem 21.

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = -1944x^6$$

Solution 21.

$$y(x) = c_1 e^{2x} + c_2 e^{-3x} + 324x^6 + 324x^5 + 1890x^4 + 2340x^3 + 4950x^2 + 3990x + 2315$$

Problem 22.

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = x + \sin(x)$$

Solution 22.

$$y(x) = c_1 e^{2x} + c_2 e^{-3x} - \frac{1}{6}x - \frac{1}{36} - \frac{1}{50} \cos(x) - \frac{7}{50} \sin(x)$$

Problem 23.

$$\frac{d^2y}{dx^2} + 4y = \sin^2(x)$$

Solution 23.

$$y(x) = c_1 \cos(2x) + c_2 \sin(2x) + \frac{1}{8} - \frac{1}{8}x \sin(2x)$$

Problem 24.

$$\frac{d^2y}{dx^2} + 4y = \sin(x)$$

Solution 24.

$$y(x) = c_1 \cos(2x) + c_2 \sin(2x) + \frac{1}{3} \sin(x)$$

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