Consider a parachutist falling through the atmosphere with downward component of velocity v. If we assume the atmospheric drag is proportional to v^2 we obtain the equation of motion

$$m\frac{dv}{dt} = mg - kv^2$$

where m is the mass of the parachutist and equipment and g is the acceleration of gravity. Since our model applies near the Earth's surface we assume g is constant, say g = 32.1 ft/sec^2, and we formulate our model in terms of the weight $W = mg$ of the parachutist and equipment. We have

$$\frac{dv}{dt} = g - \frac{gk}{W}v^2.$$

Introducing the constants

$$\alpha = \sqrt{\frac{W}{k}} \text{ and } \beta = \frac{g}{\alpha}$$

we obtain

$$\frac{dv}{dt} = \frac{\beta}{\alpha}\left(\alpha^2 - v^2\right).$$

<u>Problem 0115 – 1</u>. Solve the previous ODE with initial condition $v(0) = v_0 \geq 0$. You should obtain

$$v(t) = \alpha\,\frac{\alpha\tanh(\beta t) + v_0}{\alpha + v_0\tanh(\beta t)}$$

though your solution may appear in a different form. Compute $\lim_{t\to\infty} v(t)$.

<u>Problem 0115 – 2</u>. By integrating the result of the previous problem find a formula for the distance $s(t)$ fallen in a given time. You should obtain

$$s(t) = \frac{\alpha}{\beta}\log\left(\cosh(\beta t) + \frac{v_0}{\alpha}\sinh(\beta t)\right)$$

though your solution may appear in a different form. Here log indicates the *natural logarithm.*

<u>Problem 0115 – 3</u>. Assume $W = 260$ lbs and our parachutist falls from rest at 31,000 feet to 2,000 feet in 114 seconds with a certain drag coefficient k. At this point the parachutist opens the parachute thereby dramatically increasing k. Use the given data and the results of the previous two problems to estimate the drag coefficient k for the free–fall part of the intrepid parachutist's trajectory. Then compute v at the very moment before the parachute opens. Convert v it to mph and you will be impressed! Plot the speed $v(t)$ and estimate when in the free–fall the parachutist's speed is for all practical purposes constant (say within 1 ft/sec of the final speed). It looks like 26.2 seconds to me, but your eyesight may be better.

Note, for large t we have $\log(\cosh(\beta t)) \approx \beta t - \log 2$.