

The Earth's acceleration of gravity at sea-level  $g$  varies with latitude. Here's a short table

equator	9.77989 m/sec <sup>2</sup>	32.0862 ft/sec <sup>2</sup>
45° latitude	9.80621 m/sec <sup>2</sup>	32.1725 ft/sec <sup>2</sup>
pole	9.83210 m/sec <sup>2</sup>	32.2575 ft/sec <sup>2</sup>
conventional	9.80655 m/sec <sup>2</sup>	32.1737 ft/sec <sup>2</sup>
	9.80 – 9.81 m/sec <sup>2</sup>	32.1 – 32.2 ft/sec <sup>2</sup>

I will try to use  $g = 9.81$  m/sec<sup>2</sup> or  $g = 32.2$  ft/sec<sup>2</sup> consistently.

One ft<sup>3</sup> pure water at sea level weighs about 62.3 lb. If it is sea water though then it weighs about 64.0 lb. One ft<sup>3</sup> is 7.48126 US gallons so a 55 gallon drum has a volume of 7.3517 ft<sup>3</sup>. If the drum is submerged in fresh water it will experience a bouyant force of about 458.0 lb in fresh water or 470.5 lb in sea water.

**Problem 0117 – 1.** A 55 gallon drum containing hazardous waste, and weighing  $W = 549.8$  lb, is dropped in the ocean. Its equation of motion is

$$m \frac{dv}{dt} = mg - kv - B$$

where  $B = 470.5$  lb. is the bouyancy,  $v$  is the downward component of the velocity,  $k = 0.08$  lb-sec/ft is an experimentally determined drag coefficient,  $m$  is the mass of the drum and its contents, and  $g = 32.2$  ft/sec<sup>2</sup> is the acceleration of gravity. If  $y$  is the distance below the sea surface then  $v = \frac{dy}{dt}$  and, by the chain rule,

$$v \frac{dv}{dy} = \frac{g}{W} (W - B - kv)$$

where  $W = mg$  is the weight of the drum and its contents. Assume the drum falls from rest at the surface and solve this last ODE implicitly. You will obtain an explicit expression for  $y$  in terms of  $v$ . Then use interpolation or other means to estimate the downward speed of the drum at 200 ft, 300 ft and 400 ft (assuming the drum does not hit the bottom first).

As a check on your work note  $v = 30.166$  ft/sec at a depth of 100 ft.

**Problem 0117 – 2.** Find the equilibrium solution ( $v = v_e$  where  $v_e$  is a constant) of the ODE

$$v \frac{dv}{dy} = \frac{g}{W} (W - B - kv).$$

How can you be sure that the falling drum in the previous problem always has a speed less than  $v_e$  no matter how far it falls?

In English units the slug is the unit of mass. We have 1 slug = 14.5959 kg. When using English units it is probably more comfortable to convert to weight as we did above.