

Suppose a drug X is quickly injected into the bloodstream and that some organ in the body removes the drug from the blood at a rate proportional to the amount present. If the amount of the drug in the bloodstream is x , the initial amount is x_0 (called the *dose*), and the rate of withdrawal is μ then

$$\frac{dx}{dt} = -\mu x, \quad x(0) = x_0.$$

Let y be the amount of the drug in the organ and assume it is destroyed, transformed, metabolized or excreted at a rate proportional to the amount present. Then

$$\frac{dy}{dt} = -\nu y + \mu x, \quad y(0) = 0.$$

In general one has $\mu > \nu$ so the amount of drug X in the organ will initially increase relatively rapidly to a maximum and then decreases more or less exponentially. The maximum amount accumulated in the organ is of considerable interest, since if it is too large it may cause damage.

Problem 0127 – 1. Solve the first differential equation above. If the organ removes 65 % of the drug X in the blood in 8 hours, find μ .

Problem 0127 – 2. Given that 75 % of the drug X in the organ is destroyed in 24 hours, find ν .

Problem 0127 – 3. Solve the second differential equation equation above and find the time that a maximum amount of X accumulates in the organ. Compute the maximum amount of the drug X in the organ in terms of the original dose x_0 .

Here is a graph of $x(t)$ and $y(t)$, assuming $x_0 = 1$. You can use the graph as a rough check on your work.

