

Here are some mixing problems. These problems are not meant to be realistic. The numbers are completely off the wall.

Consider a tank initially containing a volume V_0 of brine (salt dissolved in water) of concentration ϵ_0 . Let $Q(t)$ be the amount of salt in the tank at time t . Note the initial amount of salt is given by $Q_0 = \epsilon_0 V_0$. Suppose brine of concentration ϵ_i flows into the tank at the volume rate r_i . Then salt is entering the tank at the rate $\epsilon_i r_i$. Suppose some evaporation takes place (pure water), say at the volume rate r_e . Suppose in addition to everything else the well-mixed brine solution is pumped out of the tank at the volume rate r_o . The concentration of this outflow is $\frac{Q}{V}$ where V is the current volume of brine in the tank.

We assume that r_i , r_e and r_o are constant. Then clearly

$$V = V_0 + (r_i - r_e - r_o)t$$

and

$$\frac{dQ}{dt} = \epsilon_i r_i - \frac{Q}{V} r_o, \quad Q(0) = Q_0.$$

Problem 0201 – 1. A 100 gallon tank initially contains 50 gallons of brine of concentration 1 oz salt per gallon. Brine of concentration 2 oz salt per gallon flows into the tank at 5 gallons per minute. The well-mixed solution is pumped out at 3 gallons per minute. Find the concentration of salt in the brine in the tank at the very moment of overflow.

Problem 0201 – 2. A 100 gallon tank initially contains 50 gallons of brine of concentration 1 oz salt per gallon. Brine of concentration 2 oz salt per gallon flows into the tank at 5 gallons per minute. The well-mixed solution is pumped out at 2 gallons per minute. In addition water evaporates from the tank at the rate 1 gallon per minute (hmm - we must be boiling it!). Find the concentration of salt in the brine in the tank at the very moment of overflow.

Problem 0201 – 3. Discuss the difference in the answers. In particular explain why the answer to the second problem is sane even though the concentration obtained is greater than 2.

The graph below provides a rough check of your answer.

