Mth 306 Exam | Fall 2005 | (This test requires you use the provided scantron)

Bent Petersen `306f2005-exam.tex` Test date: Noon, Tuesday, December 6, 2005 Time: 110 minutes

Instructions: $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

- *This test is a multiple-choice test. Be sure you put your name on the scantron.*
- *You must mark your answer on the provided scantron. Fill in the appropriate bubbles on the scantron very carefully.*
- *You may use one 8.5×11 inch note sheet prepared in advance. You may write on both sides of your note sheet.*
- *Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes.*
- *You may not use any books, notebooks, additional note sheets nor note cards.*
- *You are expected to have a simple scientific calculator or a modest graphics calculator available for use on this test. Calculators and other equipment may not be shared.*
- *You may use a simple graphics calculator but not a laptop computer nor any device capable of extensive symbolic manipulation (other than your own brain).*
- *There are 13 multiple-choice problems worth 12 points each.*

Be sure to enter all required information on the scantron.

Section Number: 001
Form Number: 001

Important Notes:

- Note that $\log(x)$ means the *natural logarithm* of x, sometimes denoted by $\ln(x)$. The logarithm with base 10 will be denoted by $\log_{10}(x)$, the logarithm with base 2 will be denoted by $\log_2(x)$, and so on.
- Return only the scantron. You may keep the test (and your note sheet).
- Make certain your calculator is set to radian mode.

Problem 1. The vector $\vec{v} = [-6, 1, -2]^T$ is an eigenvector of the matrix

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 1 & -3 & -7 \\ 0 & 2 & 6 \end{bmatrix}.$$

Find the corresponding eigenvalue.

A.) -1 **B.)** 4
C.) 5 **D.)** 6 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 1).*

Problem 2. The scalar 3 is an eigenvalue of the matrix

$$A = \begin{bmatrix} -9 & -7 & 7 \\ 5 & 5 & -4 \\ -7 & -5 & 6 \end{bmatrix}.$$

Find a corresponding eigenvector.

A.) $[1, 1, 0]^T$ **B.)** $[14, -13, 11]^T$
C.) $[3, -1, 2]^T$ **D.)** $[14, 13, -11]^T$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 2).*

Problem 3. The matrix

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

has real eigenvalues. Find the smallest eigenvalue.

A.) 0 **B.)** 1
C.) 2 **D.)** 3 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 3).*

Problem 4. Find the determinant $\det(A)$ where

$$A = \begin{bmatrix} a & b & c \\ 1 & 0 & -1 \\ 2 & 1 & 3 \end{bmatrix}.$$

A.) $a - b + c$ **B.)** $a - 5b + c$
C.) $a + b + c$ **D.)** $a + 5b + c$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 4).*

Problem 5. Let

$$A = \begin{bmatrix} 3 & 2 \\ -1 & a \end{bmatrix}.$$

For what value of a is A^2 diagonal?

A.) -3 **B.)** -2
C.) 2 **D.)** 3 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 5).*

Problem 6. The system of linear equations with augmented matrix

$$M = \begin{bmatrix} 1 & 2 & 3 & 3 \\ 2 & 4 & 1 & 2 \\ 3 & 6 & 1 & 6 \end{bmatrix}$$

has

A.) No solutions **B.)** A unique solution
C.) A one parameter family of solutions **D.)** A two parameter family of solutions **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 6).*

Problem 7. The system of linear equations with augmented matrix

$$M = \begin{bmatrix} 1 & 2 & 3 & 3 \\ 4 & 5 & 1 & 3 \\ 2 & 1 & -5 & -3 \end{bmatrix}$$

has

A.) No solutions **B.)** A unique solution
C.) A one parameter family of solutions **D.)** A two parameter family of solutions **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 7).*

Problem 8. The system of linear equations with augmented matrix

$$M = \begin{bmatrix} 1 & 2 & 3 & 3 \\ 4 & 5 & 1 & 3 \\ 3 & 1 & -5 & -3 \end{bmatrix}$$

has

A.) No solutions **B.)** A unique solution
C.) A one parameter family of solutions **D.)** A two parameter family of solutions **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 8).*

Problem 9. The system of linear equations with augmented matrix

$$M = \begin{bmatrix} 1 & 2 & 3 & 3 \\ 4 & 5 & 1 & 3 \\ 2 & 1 & -5 & -2 \end{bmatrix}$$

has

A.) No solutions **B.)** A unique solution
C.) A one parameter family of solutions **D.)** A two parameter family of solutions **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 9).*

Problem 10. Find the radius of convergence of the power series

$$\sum_{n=3}^{\infty} \frac{(3n)!}{3^{2n}\, n^{3n}}\, x^n.$$

A.) $3e^{-3}$ **B.)** $\frac{1}{3}e^{-3}$
C.) $3e^{3}$ **D.)** $\frac{1}{3}e^{3}$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 10).*

Problem 11. For a certain function $f(x)$ the Taylor polynomial of degree 6 about the origin is given by

$$p(x) = 2 - x + 3x^2 + 3x^4 - 6x^5 + 2x^6.$$

If

$$\left| f^{(7)}(\xi) \right| \leq 2.124$$

for each ξ in the interval $[-1/2, 1/2]$ use the Taylor remainder to estimate the maximum error

$$\max_{|x| \leq 0.5} |f(x) - p(x)|$$

in $p(x)$ on $[-1/2, 1/2]$ when viewed as an approximation of $f(x)$. Select the smallest bound $\cdots$

A.) 3.562×10^{-4} **B.)** 3.293×10^{-6}
C.) 1.112×10^{-6} **D.)** 2.781×10^{-7} **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 11).*

Problem 12. Let

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{v}_4 = \begin{bmatrix} -3 \\ 2 \\ 5 \end{bmatrix}.$$

Find the dimension of span$(\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4)$.

A.) 1 **B.)** 2
C.) 3 **D.)** 4 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 12).*

Problem 13. Let

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 8 \\ 3 \\ -1 \end{bmatrix}, \quad \vec{v}_4 = \begin{bmatrix} 7 \\ 7 \\ 6 \end{bmatrix}.$$

Find the dimension of span$(\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4)$.

A.) 1 **B.)** 2
C.) 3 **D.)** 4 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 13).*

Use the backs of the test pages for scratch work.

Enjoy your Winter break!