

Bent Petersen 306f2005-test-1.tex Test date: October 21, 2005 Time: 50 minutes

**Instructions:**  $\implies$ 

If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to enter all required information on the scantron.

Section Number: 001

Form Number: 001

- This test is a multiple-choice test. Be sure you put your name on the scantron.
- You must mark your answer on the provided scantron. Fill in the appropriate bubbles on the scantron very carefully.
- You may use one 8.5 × 11 inch note sheet prepared in advance. You may write on both sides of your note sheet.
- Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes.
- You may not use any books, notebooks, additional note sheets nor note cards.
- You are expected to have a simple scientific calculator or a modest graphics calculator available for use on this test. Calculators and other equipment may not be shared.
- You may use a simple graphics calculator but not a laptop computer nor any device capable of extensive symbolic manipulation (other than your own brain).
- There are 8 multiple-choice problems worth 12 points each.

**Important Notes:**

- Note that  $\log(x)$  means the *natural logarithm* of  $x$ , sometimes denoted by  $\ln(x)$ . The logarithm with base 10 will be denoted by  $\log_{10}(x)$ , the logarithm with base 2 will be denoted by  $\log_2(x)$ , and so on.
- Return only the scantron. You may keep the test (and your note sheet).
- Make certain your calculator is set to radian mode.

**Problem 1.** The Taylor polynomial of degree 5 of a certain function  $f(x)$  is given by

$$p(x) = -\frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{128}x^5.$$

If

$$\left| f^{(6)}(\xi) \right| \leq 6.23$$

for  $\xi$  in the interval  $[-0.5, 0.5]$  use the Taylor remainder to estimate the maximum absolute error

$$\max_{|x| \leq 0.5} |f(x) - p(x)|$$

in  $p(x)$  on  $[-0.5, 0.5]$  when viewed as an approximation of  $f(x)$ . (Select the smallest bound.)

- A.)  $9.80 \times 10^{-2}$     B.)  $8.64 \times 10^{-3}$   
 C.)  $1.37 \times 10^{-4}$     D.)  $9.72 \times 10^{-5}$

← Write letter corresponding to your answer here and mark it on the scantron (Problem 1).

**Problem 2.** The series

$$\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$$

- A.) converges by the root test    B.) diverges by the root test  
 C.) diverges by the basic div. test (0-test)    D.) converges by the integral test    E.) None of the foregoing.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 2).

**Problem 3.** Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{1 + 3^n}{5^n}$$

- A.) diverges      B.)  $\frac{27}{10}$   
C.)  $\frac{5}{2}$           D.)  $\frac{15}{4}$       E.) 5

← Write letter corresponding to your answer here and mark it on the scantron (Problem 3).

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**Problem 4.** The series

$$\sum_{n=2}^{\infty} \frac{1}{n(\log(n))^2}$$

- A.) converges by the ratio test      B.) diverges by the ratio test  
C.) converges by the integral test      D.) diverges by the integral test      E.) None of the foregoing.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 4).

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**Problem 5.** The series

$$\sum_{n=0}^{\infty} \frac{(4n + 7)^n}{(5n + 3)^n}$$

- A.) diverges by the root test      B.) converges by the root test  
C.) diverges by the integral test      D.) All tests fail      E.) None of the foregoing.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 5).

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**Problem 6.** If  $a_n > 0$  and  $b_n > 0$  for each  $n \geq 0$ , and

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 7,$$

and  $\sum_{n=0}^{\infty} b_n$  converges then

- A.)  $\sum a_n$  diverges      B.)  $\sum a_n$  converges  
C.) no conclusion is possible      D.) None of the foregoing.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 6).

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**Problem 7.** Find the radius of convergence of the power series

$$\sum_{n=3}^{\infty} \frac{n!3^n}{n^n} (x - 3)^n.$$

- A.)  $3e$       B.)  $3/e$   
C.)  $e/3$       D.)  $1/3$       E.) None of the foregoing.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 7).

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**Problem 8.** Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{n^{n+3}}{(4n+1)^n} x^{2n}.$$

- A.) 2      B.) 4  
C.)  $\frac{1}{2}$     D.)  $\frac{1}{4}$     E.) None of the foregoing.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 8).

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Use this space and the backs of the test pages for scratch work.