Mth 306 Exam | Summer 2007 | Name:

Bent Petersen `306u2007-exam.tex`

Date: 12:30 PM, August 15 2007. Location: KIDD 236. Time: 80 m in.

- Before doing anything else fill in your name in the space provided above.
- This test consists of multiple-choice problems and one or more work-out problems. Fill in the answers to the multiple-choice problems in the boxes provided. A scantron is not required. Depending on your solution methods your answers may appear in a different form from the ones provided on the test. You are expected to be able to provide the appropriate manipulations to identify the correct answer.
- For the work-out problem(s) you are expected to show some well-chosen work. Otherwise you will not receive full credit.
- You may use one 8.5×11 inch note sheet prepared in advance. You may write on both sides of your note sheet. Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes. You may not use any books, notebooks, additional note sheets nor note cards.
- You may use a simple scientific calculator or a modest graphics calculator on this test and you are expected to have one available. An overly elaborate calculator, laptop, handheld or notebook computer, or any device capable of extensive symbolic manipulation (other than your own brain) will not be allowed. Calculators and other equipment may not be shared.
- During the test be sure to check the board occasionally for corrections. Note $\log(x)$ means the natural logarithm of x.
- There are 3 multiple-choice problems worth 10 points each, 8 multiple-choice problems worth 16 points each, and 1 work-out problem(s) worth 20 points each. The total number of points is 178 points. The number of problems is 12.

Multiple-choice problems: 3 problems, 10 points each.

Problem 1. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} n!\, x^n.$$

A.) 0 **B.)** 1

C.) 2 **D.)** ∞ **E.)** None of the foregoing.

☐ *←Mark your answer here* *(Problem 1).*

Problem 2. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{1}{n!} x^n.$$

A.) 0 **B.)** 1

C.) 2 **D.)** ∞ **E.)** None of the foregoing.

☐ ←*Mark your answer here* *(Problem 2).*

Problem 3. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} x^n.$$

A.) 0 **B.)** 1

C.) 2 **D.)** ∞ **E.)** None of the foregoing.

☐ ←*Mark your answer here* *(Problem 3).*

Multiple-choice problems: 8 problems, 16 points each.

Problem 4. The infinite series

$$\sum_{n=10}^{\infty} \frac{1}{n\ \log(n)\ (\log\log(n))^2}$$

A.) converges by the ratio test **B.)** diverges by the ratio test

C.) converges by the integral test **D.)** diverges by the integral test

E.) None of the foregoing.

☐ ←*Mark your answer here* *(Problem 4).*

Problem 5. The infinite series

$$\sum_{n=10}^{\infty} \frac{1}{n\ \log(n)\ \log\log(n)}$$

A.) converges by the ratio test **B.)** diverges by the ratio test

C.) converges by the integral test **D.)** diverges by the integral test

E.) None of the foregoing.

☐ ←*Mark your answer here* *(Problem 5).*

Problem 6. Find the radius of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{2^n (n!)^3}{(2n)!(n+1)!} x^n$$

A.) 1 **B.)** 2

C.) π **D.)** $1/2$ **E.)** None of the foregoing.

☐ ←*Mark your answer here* *(Problem 6).*

Problem 7. Use the root test or the ratio test to investigate the convergence of the series

$$\sum_{n=1}^{\infty} (-1)^n \left(\frac{5n+6}{6n+5}\right)^n .$$

(In the proposed answers below r is the limiting root or limiting ratio.)

A.) converges absolutely, $r < 1$ **B.)** converges, but only conditionally

C.) diverges, $r > 1$ **D.)** test fails, $r = 1$

E.) None of the foregoing.

☐ ←*Mark your answer here* *(Problem 7).*

Problem 8. Find the radius of convergence of the series

$$\sum_{n-0}^{\infty} \frac{n^n}{n!} x^n.$$

A.) 0 **B.)** 1

C.) 2 **D.)** 3 **E.)** None of the foregoing.

☐ ←*Mark your answer here* *(Problem 8).*

Problem 9. First find the finite sum

$$\sum_{k=1}^{n} \frac{k}{(k+1)!}$$

by replacing the numerator k by $(k+1)-1$. Then use your result to compute

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)!}.$$

A.) 1 **B.)** $\mathrm{e}-2$

C.) $5\mathrm{e}^{-1}/2$ **D.)** 2 **E.)** None of the foregoing.

☐ ←*Mark your answer here* *(Problem 9).*

Problem 10. A ball is dropped onto the floor from a height of 3 feet. Each time the ball bounces it bounces to a height 5/8th of it's previous height. Find the total distance (up and down) travelled by the ball.

A.) 7 feet **B.)** 10 feet
C.) 13 feet **D.)** 16 feet **E.)** None of the foregoing.

☐ ←*Mark your answer here* *(Problem 10).*

Problem 11. Suppose we use the partial sum

$$S_{20} = \sum_{n=0}^{20} \frac{(-1)^n}{5^n}$$

to estimate the sum of the geometric series

$$S := \sum_{n=0}^{\infty} \frac{(-1)^n}{5^n}$$

We can of course compute the (absolute) error (about 1.747×10^{-15}) but we can also use the alternating series test to conveniently estimate the error. Find this error estimate (choose the closest value).

A.) 1.747×10^{-15} **B.)** 1.935×10^{-15}
C.) 2.097×10^{-15} **D.)** 2.121×10^{-15} **E.)** None of the foregoing.

☐ ←*Mark your answer here* *(Problem 11).*

Work-out problems: 1 problem(s), 20 points each.

Problem 12. A certain function f has the Taylor-Maclaurin polynomial of degree 8 given by

$$p(x) = 1 - \frac{1}{120}x^5 - \frac{1}{5040}x^7.$$

We know

$$\left| f^{(9)} \right| \leq 155 \quad \text{for } |x| \leq 1.$$

Naturally we would like to use $p(1) = 1.00813492\cdots$ as an estimate of $f(1)$. Use the Taylor remainder estimate to bound the error in $p(1)$.

Enjoy the rest of your Summer!