Some people do not read test instructions, either because they are worried about time, or because they are barely conscious immediately after entering the room for the test. It would be more productive to relax, take a deep breath and start the test in good spirits, but not everyone can do it. So here is the actual header (almost) from the test. Please read the instructions carefully before the test.

Below the sample header you may find some sample problems if I had time to compose some. The number of problems below is in no way indicative of the length of the actual test. Moreover the actual test will be multiple choice, though the problems below (if any) need not be because I didn't want to spend the time to make up "incorrect" answers.

Mth 306 Sample 1 | Summer 2007 | Name:

Bent Petersen `306u2007-sa1.tex`

Date: 12:30 PM, July 18 2007. Location: KIDD 236. Time: 80 m in.

- Before doing anything else fill in your name in the space provided above.
- This test consists of multiple-choice problems and one or more work-out problems. Fill in the answers to the multiple-choice problems in the boxes provided. A scantron is not required. Depending on your solution methods your answers may appear in a different form from the ones provided on the test. You are expected to be able to provide the appropriate manipulations to identify the correct answer.
- For the work-out problem(s) you are expected to show some well-chosen work. Otherwise you will not receive full credit.
- You may use one 8.5×11 inch note sheet prepared in advance. You may write on both sides of your note sheet. Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes. You may not use any books, notebooks, additional note sheets nor note cards.
- You may use a simple scientific calculator or a modest graphics calculator on this test and you are expected to have one available. An overly elaborate calculator, laptop, handheld or notebook computer, or any device capable of extensive symbolic manipulation (other than your own brain) will not be allowed. Calculators and other equipment may not be shared.
- During the test be sure to check the board occasionally for corrections. Note $\log(x)$ means the natural logarithm of x.
- There are 1 multiple-choice problems worth 9 points each, 0 multiple-choice problems worth 15 points each, and 8 work-out problem(s) worth 12 points each. The total number of points is 105.

The sample problems below have tentative answers provided. In some cases I may have mistyped the question, or the answer. If your answer doesn't agree with mine then check your work. Let me know if you can not reconcile your work with mine.

For additional sample problems check the old tests on my webpage.

Problem 1. Find the eigenvalues and corresponding eigenvectors for the matrix

$$\begin{bmatrix} 4 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{bmatrix}.$$

In the event that there are eigenvalue with multiplicity greater than 1 find as many linearly independent eigenvectors as possible.

Answer:

$$\begin{array}{rcl} 0 & \rightarrow & [1,-2,0]^T \\ 0 & \rightarrow & [0,-2,1]^T \\ 9 & \rightarrow & [2,1,2]^T \end{array}$$

Problem 2. Find the eigenvalues and corresponding eigenvectors for the matrix

$$\begin{bmatrix} 0 & -1 & 4 \\ 3 & 1 & -1 \\ 2 & 1 & -2 \end{bmatrix}.$$

In the event that there are eigenvalue with multiplicity greater than 1 find as many linearly independent eigenvectors as possible.

Answer:

$$\begin{array}{rcl} 0 & \rightarrow & [-1,4,1]^T \\ 2 & \rightarrow & [1,2,1]^T \\ -3 & \rightarrow & [-1,1,1]^T \end{array}$$

Problem 3. Find the eigenvalues and corresponding eigenvectors for the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 3 & 6 & 2 \end{bmatrix}.$$

In the event that there are eigenvalue with multiplicity greater than 1 find as many linearly independent eigenvectors as possible.

Answer:

$$\begin{array}{rcl} 1 & \rightarrow & [0,1,-6]^T \\ 1 & \rightarrow & \text{(none)} \\ 2 & \rightarrow & [0,0,1]^T \end{array}$$

Problem 4. Find the eigenvalues and corresponding eigenvectors for the matrix

$$\begin{bmatrix} 2 & 2 & 4 \\ 2 & -1 & 2 \\ 4 & 2 & 2 \end{bmatrix}.$$

In the event that there are eigenvalue with multiplicity greater than 1 find as many linearly independent eigenvectors as possible.

Answer:

$$\begin{array}{rcl} 7 & \rightarrow & [2,1,2]^T \\ -2 & \rightarrow & [1,-2,0]^T \\ -2 & \rightarrow & [0,-2,1]^T \end{array}$$

Problem 5. Find the eigenvalues and corresponding eigenvectors for the matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ -4 & 3 & 0 \\ 3 & 6 & 2 \end{bmatrix}.$$

In the event that there are eigenvalue with multiplicity greater than 1 find as many linearly independent eigenvectors as possible.

Answer:

$$\begin{array}{rcl} 2 & \rightarrow & [0,0,1]^T \\ 2 & \rightarrow & \text{(none)} \\ 3 & \rightarrow & [0,1,6]^T \end{array}$$

Problem 6. Find the eigenvalues and corresponding eigenvectors for the matrix

$$\begin{bmatrix} 2 & 0 & 1 \\ -4 & 3 & 0 \\ 3 & 6 & 2 \end{bmatrix}.$$

In the event that there are eigenvalue with multiplicity greater than 1 find as many linearly independent eigenvectors as possible.

Answer:

$$\begin{array}{rcl} -1 & \rightarrow & [1,1,-3]^T \\ 4+\sqrt{5}i & \rightarrow & [-1-\sqrt{5}i, 4, 3-3\sqrt{5}i]^T \\ 4-\sqrt{5}i & \rightarrow & [-1+\sqrt{5}i, 4, 3+3\sqrt{5}i]^T \end{array}$$

Problem 7. If $A = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}$ then compute the determinant $\det(AB - BA)$.

Answer: -56

Problem 8. If $A = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}$ then compute the determinant $\det(AB)$.

Answer: -56

The previous two problems illustrate the danger of jumping to unwaranted conclusions. Try a few other examples to see why.

Problem 9. If $A = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}$ then compute the trace $\operatorname{tr}(AB - BA)$.

Answer: 0

Problem 10. The vector $[1, 1, 2]^T$ is an eigenvector of the matrix

$$\begin{bmatrix} 7 & 2 & -7 \\ 10 & -1 & -7 \\ 10 & 4 & -12 \end{bmatrix}$$

Find the corresponding eigenvalue.

Answer: -5

Problem 11. One of the eigenvalues of the matrix

$$\begin{bmatrix} 7 & 2 & -7 \\ 10 & -1 & -7 \\ 10 & 4 & -12 \end{bmatrix}$$

is 2. Find a corresponding eigenvector.

Answer: $[1, 1, 1]$

I have run out of time to create sample problems. Please look at some of the old tests to get an idea what other problems you should be able to solve.