Mth 306 Exam | **Name:** **ID:**

Bent Petersen `306w2005-exam.tex` Tuesday, March 15 2005, 18:00 Time: 110 minutes.

Instructions: $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

- *This test is mostly multiple-choice but may contain some workout workout problems. You must turn in both the test and the scantron.*
- *For the multiple-choice problems you must mark your answer on the provided scantron. Fill in the appropriate bubbles on the scantron very carefully.*
- *For the workout problems you must show your work in reasonable detail on the test. Partial credit is allocated only for clear and relevant work.*
- *You may use one 8.5×11 inch note sheet prepared in advance. You may write on both sides of your note sheet.*
- *Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes.*
- *You may not use any books, notebooks, additional note sheets nor note cards.*
- *You are expected to have a simple scientific calculator available for use on this test. Calculators and other equipment may not be shared.*
- *You may use a simple graphics calculator but not a laptop computer nor any device capable of extensive symbolic manipulation (other than your own brain).*
- *There are 15 multiple-choice problems worth 8 points each and 0 work-out problems worth 20 points each.*

Be sure to enter all required information on the scantron and on this test.

Section Number: 001
Form Number: 001

Important Notes:

- Note that $\log(x)$ means the *natural logarithm* of x, sometimes denoted by $\ln(x)$. The logarithm with base 10 will be denoted by $\log_{10}(x)$, the logarithm with base 2 will be denoted by $\log_2(x)$, and so on.
- If you are taking this test in the Mathematics Learning Center you will not need a scantron. Just be sure to write the letters corresponding to your answers in the boxes provided below.

Problem 1. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{x^n}{3^{2n}}.$$

A.) 3 **B.)** 9
C.) 1/3 **D.)** 1/9 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 1).*

Problem 2. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{3^{2n}}.$$

A.) 3 **B.)** 9
C.) 1/3 **D.)** 1/9 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 2).*

Problem 3. If $A = \begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -3 \\ 2 & -2 \end{bmatrix}$ and $AB - BA = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $d =$

A.) 2 **B.)** -2
C.) 4 **D.)** -4 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 3).*

Problem 4. The system of linear equations

$$\begin{bmatrix} 2 & -3 & 3 \\ 1 & -3 & 1 \\ 1 & 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$$

has

A.) no solutions. **B.)** exactly one solution.
C.) infinitely many solutions. **D.)** a 2 dimensional solution space. **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 4).*

Problem 5. The system of linear equations (Note: same coefficient matrix as in the previous problem)

$$\begin{bmatrix} 2 & -3 & 3 \\ 1 & -3 & 1 \\ 1 & 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

has

A.) no solutions. **B.)** exactly one solution.
C.) infinitely many solutions. **D.)** a 2 dimensional solution space. **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 5).*

Problem 6. The system of linear equations (Note: NOT the same coefficient matrix as in the previous problem)

$$\begin{bmatrix} 2 & -3 & 3 \\ 1 & -3 & 1 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

has

A.) no solutions. **B.)** exactly one solution.
C.) infinitely many solutions. **D.)** a 2 dimensional solution space. **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 6).*

Problem 7. Compute the determinant of the matrix

$$A = \begin{bmatrix} 1 & k & 3 \\ 3 & 2 & 1 \\ 2 & k & 2 \end{bmatrix}.$$

A.) $k - 5$ **B.)** -8

C.) $8k - 16$ **D.)** $4k - 8$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 7).*

Problem 8. Let

$$A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 1 & k \\ 3 & 2 & 1 \end{bmatrix}.$$

For what values of k does the system of linear equations $A\vec{x} = \vec{0}$ have infinitely many solutions?

A.) $k = 1/2$ **B.)** $k = 1$

C.) $k = 3/2$ **D.)** all k **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 8).*

Problem 9. Let

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 3 \end{bmatrix}.$$

Find the eigenvalues of A. Then find an eigenvector corresponding to the smallest eigenvalue.

A.) $[0, 1, 2]$ **B.)** $[1, 0, 0]$

C.) $[0, -2, 1]$ **D.)** $[2, -1, 0]$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 9).*

Problem 10. The matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

has real eigenvalues. One of its eigenvalues is

A.) 0 **B.)** 2

C.) 4 **D.)** 6 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 10).*

Problem 11. The matrix (Note: A is the same matrix as in the previous problem)

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

has real eigenvalues. One of its eigenvalues is 3. Find a corresponding eigenvector (T indicates the transpose).

A.) $[1, 1, 0]^T$ **B.)** $[1, 1, -1]^T$

C.) $[1, 1, 1]^T$ **D.)** $[0, 1, -1]^T$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 11).*

Problem 12. The vector $[1, 1, 2]^T$ is an eigenvector of the matrix

$$A = \begin{bmatrix} -3 & -3 & 5 \\ -4 & -2 & 5 \\ -4 & -6 & 9 \end{bmatrix}.$$

Find the corresponding eigenvalue.

A.) -1 **B.)** 1

C.) 3 **D.)** 4 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 12).*

Problem 13. Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & 1 & -1 \end{bmatrix}.$$

A.) $3, 2, -1$ **B.)** $-3, -2, -1$

C.) $3, 4, -1$ **D.)** $-3, -5, -1$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 13).*

Problem 14. The matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & 1 & -1 \end{bmatrix}$$

(of the previous problem) has an eigenvalue equal to -1. Find a corresponding eigenvector.

A.) $[3, -6, 1]^T$ **B.)** $[0, 4, 1]^T$

C.) $[0, 0, 1]^T$ **D.)** $[1, 0, 0]^T$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 14).*

Problem 15. The rows of the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 3 & -1 \\ 3 & 5 & 1 \end{bmatrix}$$

A.) form a basis of $\mathbb{R}^3$ **B.)** span (or generate) $\mathbb{R}^3$

C.) span a proper subspace of $\mathbb{R}^3$ **D.)** are linearly independent **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 15).*

Use this page and the backs of all the pages for scratch work.