MTH 311 ASSIGNMENT 2 *Due: Oct 17, 1997**

Please turn in neat carefully written solutions to the problems. You should try to write good proofs. You may discuss the problems with anyone for the purpose of obtaining ideas and clarification. You are expected however to produce and to write–up your own solutions.

Problem 3. Let $(a_n)_{n\geq 1}$ be a sequence of complex numbers. Show that the sequence $(a_n)_{n\geq 1}$ converges to a if and only if each subsequence of $(a_n)_{n\geq 1}$ has a subsequence which converges to a.

Give an example of a divergent sequence $(b_n)_{n\geq 1}$ such that each subsequence of $(b_n)_{n\geq 1}$ has a subsequence which converges.

The first part of this problem is an exercise in the text. It is best done by thinking about the contrapositive. The second part is easy once you realize what to do. (Isn't that frequently the case?)

Problem 4. Let $0 < c < 1$. Prove $\lim_{n\to\infty} c^n = 0$.

Use minimal information here. In particular do not make use of logarithms or the exponential as we have not discussed them yet.

Problem 5. Let $0 < c < 1$. Let $(a_n)_{n\geq 1}$ be a sequence of complex numbers such that

$$| a_{n+1} - a_n | \leq c^n$$

for each $n \in \mathbb{N}$. Prove that $(a_n)_{n\geq 1}$ is a Cauchy sequence.

Do this problem directly by using the triangle inequality and the previous problem. Do not use anything you learned about convergence of sequences and series in Calculus (except as a guide).

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