# Mth 351 Exam | Name: ID:

Bent Petersen `351f2001_exam.tex` Dec 5, 2001 Time: 110 minutes.

**Instructions:** $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to write your name in the space above.

- *You may use one note-sheet prepared in advance. You must put your name on your note-sheet, but do not turn in your note-sheet. Your note-sheet must be letter size,* $8.5 \times 11$ *inches, or A4 paper,* $21 \times 29.7$ *cm, or smaller. You may write on both sides of your note-sheet.*
- *Note-sheets may not be shared. If you do not bring a note-sheet you will have to do without any help notes.*
- *You may not use any books, notebooks nor additional note-sheets.*
- *You may use a calculator. Calculators and other equipment may not be shared.*
- *For multiple-choice problems indicate your choice in the answer box provided. You need not show any work nor offer any explanations for your answer. If you need to do some work, you may do it in the space provided, if any, or on the back of the examination sheets, but your work will not be graded.* ***You will be graded only on the letter you select and put in the provided answer box.*** *Note this test does not use a scantron.*
- *Use the backs of the examination sheets for scratch work.*

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*Please note* $\log(x)$ *means the natural logarithm of* $x$ *on this test.*

**Note:** Some problems may have more than one technically "correct" answer. In that case the best answer receives full credit, whereas correct, but not best, answers receive partial credit. For example, a small integer greater than 5 is probably 6, whereas $10^9$ is also technically correct, but probably not best (depending on what other selections are available). Problems with partial credit are indicated by multiple points, e.g., 20,5.

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**Problem 1.** *(20 points if correct, 0 points if wrong).* Suppose we have a binary computer with a mantissa of length 12. Suppose we chop numbers to store them and suppose we do not pack the most significant bit. Assuming we do not incur overflow, what is the roundoff error in storing $\dfrac{47}{3}$?

**A.)** $\frac{1}{128}$ **B.)** $\frac{1}{384}$
**C.)** $\frac{1}{1536}$ **D.)** $\frac{1}{3072}$ **E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 1.*

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**Problem 2.** *(20 points if correct, 0 points if wrong).* Compute the Taylor polynomial of degree 4 with center at the origin for $f(x) = \mathrm{e}^{\cos(x)-1}$.

**A.)** $1 - \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{6}x^4$ **B.)** $1 - \frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{1}{6}x^4$
**C.)** $1 - \frac{1}{2}x^2 + \frac{1}{6}x^4$ **D.)** $1 - x^2 + 4x^4$ **E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 2.*

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**Problem 3.** *(20 points if correct, 0 points if wrong).* The degree 3 polynomial $p(x)$ satisfies $p(0) = -5, p(1) = -6, p(2) = -5, p(3) = 4, p(4) = 27$. Therefore $p(x)$ has a root in the interval

**A.)** $[0, 1]$ **B.)** $[1, 2]$
**C.)** $[2, 3]$ **D.)** $[3, 4]$ **E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 3.*

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**Problem 4.** *(20,10 points if correct, 0 points if wrong).* Let $q$ be a continuously differentiable function defined on $[-1, 1]$. Suppose $0 \le q(x) \le 1$ and $|\, q\prime(x) \,| \le \frac{110}{111}$, for each $x$ with $-1 \le x \le 1$. Then

**A.)** $q$ has a unique fixed point $\alpha$ in $[-1, 1]$
**B.)** $q$ has a unique fixed point $\alpha$ in $[-1, 1]$ and $-1 \le \alpha < 0$
**C.)** $q$ has a unique fixed point $\alpha$ in $[-1, 1]$ and $0 \le \alpha \le 1$
**D.)** $q$ has no fixed points in $[-1, 1]$
**E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 4.*

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**Problem 5.** *(20 points if correct, 0 points if wrong).* The matrix $A = \begin{bmatrix} 6 & -4 & 6 \\ -6 & 14 & -21 \\ -6 & 10 & -15 \end{bmatrix}$ has the eigenvector $v = [1, 4, 2]^T$. Find the corresponding eigenvalue.

**A.)** 3 **B.)** 2
**C.)** 1 **D.)** 0 **E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 5.*

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**Problem 6.** *(20,10 points if correct, 0 points if wrong).* Let $A$ be a symmetric $n \times n$ real matrix. Then

**A.)** $A$ has real eigenvalues and is diagonizable
**B.)** $A$ is diagonizable if $A$ has distinct eigenvalues
**C.)** $A$ is diagonizable only if $A$ has distinct eigenvalues
**D.)** $A$ is diagonizable
**E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 6.*

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**Problem 7.** *(20 points if correct, 0 points if wrong).* Let $G_2$ be Gauss quadrature on $[-1, 1]$ of order 2, that is, $G_2(f) = f(-\frac{1}{\sqrt{3}}) + f(\frac{1}{\sqrt{3}})$. If $p(x)$ is any polynomial of degree 3 or less, and we use $G_2(p(x) + x^4)$ to estimate $\int_1^{-1} \left(p(x) + x^4\right)\, dx$ what error do we make?

**A.)** 0 **B.)** $\frac{2}{9}$
**C.)** $\frac{8}{45}$ **D.)** $\frac{-2}{45}$ **E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 7.*

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**Problem 8.** *(20 points if correct, 0 points if wrong).* Let $p(x)$ be a polynomial of degree 3 with Newton divided differences $p[1] = 1$, $p[1, 2] = 1$, $p[1, 2, 3] = -\frac{1}{2}$ and $p[1, 2, 3, 5] = \frac{1}{6}$. Compute $p(5)$.

**A.)** 4 **B.)** 3
**C.)** 2 **D.)** 1 **E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 8.*

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