

## Midpoint Method for Estimating Roots

### Mth 351 Sept 30 2001 Maple 6

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Filename: 351f2001\_midpoint.mws

Bisection method for approximating a root

```
> restart; Digits:=16;
```

```
> midp:=L->(L[1]+L[2])/2;
```

$$midp := L \rightarrow \frac{1}{2}L_1 + \frac{1}{2}L_2$$

```
> bisect:=proc(f,L)
```

```
>   local a,b,c;
```

```
>   a:=L[1];b:=L[2];
```

```
>   c := (a+b)/2;
```

```
>   if f(a) = 0 then
```

```
>     return [a,c]
```

```
>   elif f(c) = 0 then
```

```
>     return [a,c];
```

```
>   elif f(b) = 0 then
```

```
>     return [c,b];
```

```
>   elif not signum(f(a)) = signum(f(b)) then
```

```
>     if signum(f(a)) = signum(f(c)) then
```

```
>       return [c,b];
```

```
>     else
```

```
>       return [a,c];
```

```
>     fi;
```

```
>   else
```

```
>     return FAIL;
```

```
>   fi;
```

```
> end;
```

Here is a sample function from the text

```
> f:=x->x-x^(1/3)-2;
```

$$f := x \rightarrow x - x^{(1/3)} - 2$$

Let's try to locate the root with an error no bigger than  $10^{-8}$

```
> L:=[1,5];
```

$$L := [1, 5]$$

```
> while L[2]-L[1] > 2*10^(-8) do L:=bisection(f,L): od: L;
rf:=evalf(midp(L));
```

$$\left[ \frac{236315791}{67108864}, \frac{14769737}{4194304} \right]$$

$$rf := 3.521379701793194$$

Let's check with Maple's solver:

```
> rftrue:=fsolve(f(x)=0,x); err:=rftrue-rf;
rftrue := 3.521379706804568
err := .5011374 10^-8
```

Here's another example from the text

```
> g:=x->x-12*x^(1/3)+12;
```

$$g := x \rightarrow x - 12x^{(1/3)} + 12$$

```
> L:=[0,5];
```

$$L := [0, 5]$$

```
> while L[2]-L[1] > 2*10^(-6) do L:=bisection(g,L): od: L;
rg1:=evalf(midp(L));
```

$$\left[ \frac{5825855}{4194304}, \frac{1456465}{1048576} \right]$$

$$rg1 := 1.388992667198181$$

```
> L:=[15,25];
```

$$L := [15, 25]$$

```
> while L[2]-L[1] > 2*10^(-6) do L:=bisection(g,L): od: L;
rg2:=evalf(midp(L));
```

$$\left[ \frac{44511655}{2097152}, \frac{89023315}{4194304} \right]$$

$$rg2 := 21.22481167316437$$

```
> rg1true:=fsolve(g(x)=0,x); err:=rg1true-rg1;
rg1true := 1.388992759956590
```

$$err := .92758409 10^{-7}$$

```
> rg2true:=fsolve(g(x)=0,x=10..infinity); err:=rg2true-rg2;
rg2true := 21.22481166331540
```

$$err := -.984897 10^{-8}$$

Now let's find the square root of 2 by bisection with an error no bigger than  $10^{-9}$

```

[ > h:=x->x^2-2;
                                      $h := x \rightarrow x^2 - 2$ 
[ > L:=[1,2];
                                      $L := [1, 2]$ 
[ > while L[2]-L[1] > 2*10^(-9) do L:=bisect(h,L): od: L;
  rh:=evalf(midp(L));
                                      $\left[ \begin{array}{cc} 189812531 & 759250125 \\ 134217728 & 536870912 \end{array} \right]$ 
                                      $rh := 1.414213561452925$ 
[ > err:=evalf(sqrt(2)-rh);
                                      $err := .920170 \cdot 10^{-9}$ 
[ >

```