Mth 351 Assignment 1 Fall 2002 | Due: Oct 16, 2002

Bent Petersen 351f2002_asg01.tex

Recall the Newton-Raphson iteration for approximating a root of the function f

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

Let $a > 0$. If we apply the Newton-Raphson method to the function $f(x) = x^2 - a$ we obtain, for reasonable x_0, a sequence converging to $\sqrt{a}$,

$$x_{n+1} = \frac{x_n^2 + a}{2x_n}.$$

In particular if we take $a = 2$ and $x_0 = 1$ we obtain the sequence

$$1, \frac{3}{2}, \frac{17}{12}, \frac{577}{408}, \frac{665857}{470832}, \frac{886731088897}{627013566048}, \ldots$$

which converges rapidly to $\sqrt{2}$.

Problem 1. Compute the error in each of terms in the sequence above. Does the convergence appear to be quadratic, or not? Explain your conclusion?

There are numerous modifications of Newton's method, for example, we have the Newton-Halley iteration for approximating a root of the function f

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \left(1 - \frac{f(x_n)f''(x_n)}{2\left(f'(x_n)\right)^2}\right)^{-1}.$$

Problem 2. Apply the Newton-Halley method to the function $f(x) = x^2 - a$ to obtain a sequence approximating $\sqrt{a}$ (be sure to simplify your result).

Problem 3. Use the result of the previous problem to obtain a sequence converging to $\sqrt{2}$. Compute the errors. Is the convergence quadratic or faster? What does the rate of convergence appear to be? Explain.

You may use a calculator, Matlab, Maple, a spreadsheet, or just about anything else. You may have some roundoff problems in computing some of the errors above.

If you elect to use a spreadsheet you may find useful the (ancient) book

William J. Orvis, *1-2-3 for Scientists and Engineers*, Sybex, San Francisco, 1987.

Rules. You may talk to anyone and get help wherever you can for any assignment, but at some point you must write up your work by yourself.

You should answer in complete reasonably correct English sentences. You should explain briefly how your results where obtained. Make an effort to communicate rather than just turning in numbers.

If you write some code and ask me to help to debug your code, you will get a well-deserved intense frown. Life's too short to debug my own code, let alone others' code.