Mth 351 Exam | Name: ID:

Bent Petersen `351f2002_exam.tex` Thursday Dec 12, 2002 Time: 110 minutes.

Instructions: $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to write your name in the space above.

- *You may use one <u>note-sheet</u> prepared in advance. You must put your name on your note-sheet, but <u>do not</u> turn in your note-sheet. Your note-sheet must be letter size,* 8.5×11 *inches, or A4 paper,* 21×29.7 *cm, or smaller. You may write on both sides of your note-sheet.*
- *Note-sheets may not be shared. If you do not bring a note-sheet you will have to do without any help notes.*
- *You may not use any books, notebooks nor additional note-sheets.*
- *You may use a calculator. Calculators and other equipment may not be shared.*
- *<u>For work-out problems</u> sketch your work neatly. Highlight your answer by drawing a frame around it. Scratch out irrelevant or incorrect work so it will be clear what you are submitting as a solution. Give exact answers when possible. Simplify your answer when reasonable to do so. Partial credit will be assigned only for relevant, clear, correct, legible work. If you do not show some relevant work or explain your solution, your grade may be 0.*
- *<u>For multiple-choice problems</u> indicate your choice in the answer box provided. <u>You need not show any work nor offer</u> any explanations for your answer. If you need to do some work, you may do it in the space provided, if any, or on the back of the examination sheets, but your work will not be graded.* ***You will be graded only on the letter you select and put in the provided answer box.*** *<u>Note this test does not use a scantron.</u>*
- *Use the backs of the examination sheets for scratch work.*

Please note $\log(x)$ *means the natural logarithm of* x *on this test.*

Problem 1. *(30 points).* A lower triangular matrix L is said to be *unit lower triangular* if its diagonal entries are all ones. Thus a 2×2 unit lower triangular matrix L is of the form

$$L = \begin{bmatrix} 1 & 0 \\ x & 1 \end{bmatrix}.$$

A basic fact is: *Let A be a $n \times n$ symmetric positive definite matrix. Then there exits a unit lower triangular matrix L and a diagonal matrix D with positive diagonal entries such that $A = LDL^T$.*

If

$$A = \begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix}$$

then find a unit lower triangular matrix L and and a diagonal matrix D such that $A = LDL^T$.

Problem 2. *(30 points).* Determine a, b and c such that the quadrature formula

$$Q(f) = a\,f(0) + b\,f\left(\frac{1}{2}\right) + c\,f(1)$$

is exact for the integral

$$\int_0^1 f(x)\,x^3\;\mathrm{d}x$$

if $f(x)$ is a polynomial of degree ≤ 2, that is, is exact for $f(x) = 1$, $f(x) = x$ and $f(x) = x^2$.

Find the error in $Q(x^4)$, as an approximation to $\displaystyle\int_0^1 x^7\;\mathrm{d}x$.

Problem 3. *(30 points).* If we approximate the integral

$$\int_a^b f(x)\ \mathrm{d}x$$

of a certain function f on a certain interval $[a, b]$ by the compound trapezoidal rule with n subintervals, $T(n)$, we obtain

$$T(24) = 0.8032593904, \quad T(48) = 0.8043979719, \quad T(96) = 0.8046819071$$

Use this information to compute the Simpson's rule estimates with n subintervals, $S(n)$, for $n = 48$ and $n = 96$.

Problem 4. *(30 points).* Find the interpolation polynomial of degree ≤ 3 through the points $(1, 2)$, $(2, 2)$, $(3, 6)$ and $(5, 2)$.

You may use the monomial basis, Lagrange basis or Newton basis, as you please.

Problem 5. *(30 points).* Let

$$s(x) = \begin{cases} 12 - 2\,x + a\,x^2 - 3\,x^3 & \text{if} \quad -1 \le x < 0 \\ 12 - 2\,x + b\,x^2 + 3\,x^3 & \text{if} \quad 0 \le x \le 1 \end{cases} .$$

For what values of a and b is $s(x)$ a *natural* cubic spline with knots at -1, 0 and 1.

Problem 6. *(30 points).* Find the polynomial

$$p(x) = a + b\,x + x^2$$

which, in the sense of least squares, best fits the data points

$$(1.0,\, -1.1), \quad (2.0,\, -0.9), \quad (3.0,\, 1.0), \quad (4.0,\, 4.8).$$

Problem 7. *(30 points).* Consider the system

$$\begin{aligned} 3\,x - y &= 3 \\ -x + 5\,y &= 5 \end{aligned}$$

The solution to this system is clearly $x = \dfrac{10}{7}$ and $y = \dfrac{9}{7}$. Suppose we perform a Jacobi iteration with initial guess $x = 1$ and $y = 2$. Compute the next 3 Jacobi iterates. Does the Jacobi iteration appear to converge to the solution?

Problem 8. *(30 points).* Write a brief technical essay, in decent English, describing the topic in Mth 351 which you most enjoyed. Your essay should contain some relevant technical facts or equations.

Additional test policies for this class are provided on my web page `http://www.onid.orst.edu/~peterseb`.

Use the space below for your scratch work.

Please do not write in the boxes to the right. They are for your grades.
Do not be concerned if there are more boxes than problems.

	Letter Grade ☐ *This test only* ☐ *Cummulative*

1	2	3	4	5	6	7	8	9	10	Total

Note: There are 8 problems for a total of 240 points.