Mth 351 Midterm | Name: ID:

Bent Petersen `351f2002_midterm.tex` Nov 8, 2002 Time: 50 minutes.

Instructions: $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to write your name in the space above.

- *You may use one* <u>*note-sheet*</u> *prepared in advance. You must put your name on your note-sheet, but* <u>*do not*</u> *turn in your note-sheet. Your note-sheet must be letter size,* 8.5×11 *inches, or A4 paper,* 21×29.7 *cm, or smaller. You may write on both sides of your note-sheet.*
- *Note-sheets may not be shared. If you do not bring a note-sheet you will have to do without any help notes.*
- *You may not use any books, notebooks nor additional note-sheets.*
- *You may use a calculator. Calculators and other equipment may not be shared.*
- <u>*For work-out problems*</u> *sketch your work neatly. Highlight your answer by drawing a frame around it. Scratch out irrelevant or incorrect work so it will be clear what you are submitting as a solution. Give exact answers when possible. Simplify your answer when reasonable to do so. Partial credit will be assigned only for relevant, clear, correct, legible work. If you do not show some relevant work or explain your solution, your grade may be 0.*
- <u>*For multiple-choice problems*</u> *indicate your choice in the answer box provided. You need not show any work nor offer any explanations for your answer. If you need to do some work, you may do it in the space provided, if any, or on the back of the examination sheets, but your work will not be graded.* ***You will be graded only on the letter you select and put in the provided answer box.*** <u>*Note this test does not use a scantron.*</u>
- *Use the backs of the examination sheets for scratch work.*

Please note $\log(x)$ *means the natural logarithm of* x *on this test.*

Problem 1. *(25 points).* Consider the polynomial

$$p(x) = x^7 - 3\,x + 1.$$

Explain how you know this polynomial has a root in the interval $[0, 1]$. If you use an initial "guess" of 0 and apply Newton's method just once what estimate do you get for the root?

Problem 2. *(25 points).* Consider the polynomial

$$p(x) = x^7 - 3\,x + 1$$

again. Suppose we use initial "guesses" of 0 and $\frac{1}{2}$ for a root. Use the secant method to obtain a new estimate of a root.

Problem 3. *(25 points).* Assume we wish to solve the system of three linear equations $A\vec{x} = \vec{b}$ in three variables and we happen to have available the LU decomposition of A, say $A = LU$. We solve the system $L\vec{y} = \vec{b}$ by forward substitution and find $\vec{y} = [3, 4, -15]^T$. If

$$U = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

then find $\vec{x}$.

Problem 4. *(25 points).* The central symmetric 3–point formula with step size h for estimating the derivative of a function f is

$$f'(a) \approx \frac{f(a+h) - f(a-h)}{2h}.$$

The error is $\mathcal{O}(h^2)$. The central symmetric 3–point formula with step size h for estimating the derivative of a function f is

$$f'(a) \approx \frac{-2f(a+2h) + 16f(a+h) - 16f(a-h) + 2f(a-2h)}{24h}.$$

The error is $\mathcal{O}(h^4)$. Given the table

x	0.240	0.340	0.440	0.540	0.640	0.740	0.840
$f(x)$	0.238	0.333	0.426	0.514	0.597	0.674	0.745

and using the smallest step size supported by the data in the table use both the central symmetric formulæ above to obtain two estimates of the derivative $f'(0.540)$.

Problem 5. *(25 points).* For a certain nonlinear spring Hooke's law has to be modified to state $F = kx + ax^3$, where F is the force (in ounces) used to stretch the spring, x is the increase in its length (in inches) and k is Hooke's constant. Determine the constants k and a which yield the best leastsquares fit to the following experimental data.

x	0.200	0.400	0.600	0.800	1.000
F	0.040	0.076	0.107	0.129	0.141

Problem 6. *(25 points).* The system of equations

$$\begin{cases} x^3 + y^3 - 3xy = 1 \\ x^2 + y^2 = 1 \end{cases}$$

has 4 solutions. The solutions are symmetric about the line $y = x$. There are 2 trivial solutions, $(0, 1)$ and $(1, 0)$. Because of the symmetry, to find the remaining 2 solutions, we need find only one solution. Using a contour plot of the equations we estimate there is a solution near the point $(0.5, -0.8)$. Apply Newton's method just once with $(0.5, -0.8)$ as the initial "guess" to obtain an estimate of the solution. Report your answer to at least 5 decimal places.

Additional test policies for this class are provided on my web page `http://www.onid.orst.edu/~peterseb`.

Use the space below for your scratch work.

Please do not write in the boxes to the right. They are for your grades.
Do not be concerned if there are more boxes than problems.

	Letter Grade ☐ *This test only* ☐ *Cummulative*

1	2	3	4	5	6	7	8	9	10	Total

Note: There are 6 problems for a total of 150 points.