Mth 351 Test 1 | Fall 2005 | (This test requires you use the provided scantron)

Bent Petersen `351f2005-test-1.tex` Test date: October 19 2005 Time: 50 minutes

Instructions: $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to enter all required information on the scantron.

Section Number: 001
Form Number: 001

- *This test is a multiple-choice test. Be sure you put your name on the scantron.*
- *You must mark your answer on the provided scantron. Fill in the appropriate bubbles on the scantron very carefully.*
- *You may use one 8.5×11 inch note sheet prepared in advance. You may write on both sides of your note sheet.*
- *Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes.*
- *You may not use any books, notebooks, additional note sheets nor note cards.*
- *You are expected to have a simple scientific calculator or a modest graphics calculator available for use on this test. Calculators and other equipment may not be shared.*
- *You may use a simple graphics calculator but not a laptop computer nor any device capable of extensive symbolic manipulation (other than your own brain).*
- *There are 8 multiple-choice problems worth 12 points each.*

Important Notes:

- Note that $\log(x)$ means the *natural logarithm* of x, sometimes denoted by $\ln(x)$. The logarithm with base 10 will be denoted by $\log_{10}(x)$, the logarithm with base 2 will be denoted by $\log_2(x)$, and so on.
- Return only the scantron. You may keep the test (and your note sheet).
- Make certain your calculator is set to radian mode.

Problem 1. The Taylor polynomial of degree 5 of a certain function $f(x)$ is given by

$$p(x) = -\frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{128}x^5.$$

If

$$\left| f^{(6)}(\xi) \right| \leq 6.23$$

for ξ in the interval $[-0.5, 0.5]$ use the Taylor remainder to estimate the maximum absolute error in $p(x)$ on $[-0.5, 0.5]$ when viewed as an approximation of $f(x)$. (Select the smallest bound.)

A.) 9.80×10^{-2} **B.)** 8.64×10^{-3}
C.) 1.37×10^{-4} **D.)** 9.72×10^{-5}

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 1).*

Problem 2. We approximate a certain function $f(x)$ by an n^{th} degree Taylor polynomial about the origin. If the absolute value of the k^{th} derivative of $f(x)$ on the interval $[-1, 1]$ is bounded by 2^k for each k how large should we choose n in order to estimate $f(0.5)$ with an error no larger than 0.0015? (Select the smallest n that works.)

A.) 4 **B.)** 5
C.) 6 **D.)** 7 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 2).*

$$
\begin{aligned}
f'(a) &= \frac{f(a+h)-f(a)}{h} + \bigcirc(h)\\
f'(a) &= \frac{f(a+h)-f(a-h)}{2h} + \bigcirc(h^2)\\
f''(a) &= \frac{f(a+h)-2f(a)+f(a-h)}{h^2} + \bigcirc(h^2)\\
f'''(a) &= \frac{f(a+2h)-2f(a+h)+2f(a-h)-f(a-2h)}{2h^3} + \bigcirc(h^2)
\end{aligned}
$$

Problem 3. A popular one-sided numeric differentiation formula is given by

$$f'(a) \sim \frac{-3f(a)+4f(a+h)-f(a+2h)}{2h}.$$

The (truncation) error here is of order $\mathcal{O}(h^p)$ for a certain p. By expanding the terms in the numerator in suitable Taylor polynomials (or some other way) determine p.

A.) $p = 1$ **B.)** $p = 2$

C.) $p = 3$ **D.)** $p = 4$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 3).*

Problem 4. The function f has the values shown in the table below.

x	0	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000
$f(x)$	2.802	2.873	2.911	3.109	3.002	2.999	2.981	2.786	2.544

Use a second order symmetric estimate with the smallest possibe step size to estimate $f'(0.500)$.

A.) -0.440 **B.)** -0.880

C.) -0.024 **D.)** -0.856 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 4).*

Problem 5. For this problem assume a machine using a normalized binary floating point representation for real numbers. Assume that the mantissa occupies 12 bits (not very useful!), there is no packing trickery, there are no reserved bit patterns for denormals, NaN's nor infinities, chopping is used and enough bits are available for the exponent that we will not have to worry about overflow nor underflow. Find the exact roundoff error incurred in storing $\frac{29}{15}$ and express the error in decimal. Select the closest number below.

A.) 9.44×10^{-4} **B.)** 4.72×10^{-4}

C.) 4.56×10^{-4} **D.)** 2.28×10^{-4} **E.)** 1.14×10^{-4}

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 5).*

Problem 6. The function f defined by $f(x) = \cos(2x) - \sin(3x)$ has a root in the interval $[0, \pi/4]$ (because it changes sign on the interval). Take the initial "guess" to be $x_0 = 0.5$ and compute the Newton iterate x_2. Choose the closest number from the list below.

A.) 0.3168 **B.)** 0.3142
C.) 0.3126 **D.)** 0.7412 **E.)** 0.4903

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 6).*

Problem 7. The Čebyšev root iteration is a modified Newton method given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\left(1 + \frac{1}{2}\,\frac{f(x_n)f''(x_n)}{f'(x_n)^2}\right).$$

If we apply this method to $f(x) = x^2 - 2$ with $x_0 = 1.0$ we see the following errors in the iterates: 4.0×10^{-1}, 3.9×10^{-2}, 1.6×10^{-5}, 1.0×10^{-15}, 2.8×10^{-46}, 5.4×10^{-138},$\cdots$. Based on this evidence you would estimate the order of Čebyšev's method to be

A.) 2 **B.)** 3
C.) 4 **D.)** 5 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 7).*

Problem 8. Newton illustrated his method of approximating roots in 1669 by approximating a root of the polynomial $x^3 - 2x - 5$. If we take $x_0 = 2.0$ then $x_2 =$?? (Select the closest number.)

A.) 1.90 **B.)** 2.10
C.) 2.09455148 **D.)** 2.09456812

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 8).*

Use the backs of the test pages for scratch work.