Mth 351 Test 2 | Fall 2005 | (This test requires you use the provided scantron)

Bent Petersen `351f2005-test-2.tex` Test date: xxxx 2005 Time: xx minutes

Instructions: $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to enter all required information on the scantron.

Section Number: 001
Form Number: 001

- *This test is a multiple-choice test. Be sure you put your name on the scantron.*
- *You must mark your answer on the provided scantron. Fill in the appropriate bubbles on the scantron very carefully.*
- *You may use one 8.5×11 inch note sheet prepared in advance. You may write on both sides of your note sheet.*
- *Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes.*
- *You may not use any books, notebooks, additional note sheets nor note cards.*
- *You are expected to have a simple scientific calculator or a modest graphics calculator available for use on this test. Calculators and other equipment may not be shared.*
- *You may use a simple graphics calculator but not a laptop computer nor any device capable of extensive symbolic manipulation (other than your own brain).*
- *There are xx multiple-choice problems worth xx points each.*

Important Notes:

- Note that $\log(x)$ means the *natural logarithm* of x, sometimes denoted by $\ln(x)$. The logarithm with base 10 will be denoted by $\log_{10}(x)$, the logarithm with base 2 will be denoted by $\log_2(x)$, and so on.
- Return only the scantron. You may keep the test (and your note sheet).
- Make certain your calculator is set to radian mode.

Problem 1. Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ where $a_0, a_1, \cdots, a_n$ are constants, and let $x_0, x_1, \cdots, x_n$ be $n+1$ distinct points. Compute the n^{th} order Newton divided difference $f[x_0, x_1, \cdots, x_n]$.

A.) $n!\, a_n$ **B.)** a_n
C.) 0 **D.)** $a_n(x_0 + x_1 + \cdots + x_n)$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 1).*

Problem 2. For a certain function $f(x)$ we know the NEWTON divided differences: $f[-1] = 2$, $f[-1,1] = 1$, $f[-1,1,2] = -2$, $f[-1,1,2,5] = 2$. Let $P_3(x)$ the interpolation polynomial for $f(x)$ of degree at most 3 with nodes at -1, 1, 2, 5. Suppose we use $P_3(0) = 9$ to estimate $f(0)$. Find a good upper bound for the absolute error in the estimate $P_3(0)$ of $f(0)$ given that $|f^{(4)}(\xi)| \leq 0.024$ for all ξ in the interval $[-1, 5]$. (Select the smallest upper bound from the following list.)

A.) 0.001 **B.)** 0.009
C.) 0.010 **D.)** 0.240 **E.)** 10^{18}

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 2).*

Problem 3. Find the interpolation polynomial $p(x)$ of degree ≤ 3 through the points $(-1, 14)$, $(1, 4)$, $(2, -1)$, $(5, -16)$.

A.) $\frac{1}{72}x^3 - \frac{1}{36}x^2 - \frac{361}{72}x + \frac{325}{36}$ **B.)** $-\frac{1}{72}x^3 + \frac{1}{36}x^2 - \frac{359}{72}x + \frac{323}{36}$

C.) $-\frac{1}{36}x^3 + \frac{1}{18}x^2 - \frac{179}{36}x + \frac{161}{18}$ **D.)** $\frac{1}{36}x^3 - \frac{1}{18}x^2 - \frac{181}{36}x + \frac{163}{18}$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 3).*

Problem 4. Let T_n be the compound trapezoidal estimate of $\int_1^3 f(x)\,\mathrm{d}x$ with n subintervals, for a certain function f. Let S_n be the compound Simpson's estimate of $\int_1^3 f(x)\,\mathrm{d}x$ with n subintervals. Suppose $T_{05} = 2.03639$, $T_{10} = 1.97381$ and $T_{20} = 1.95838$. Compute S_{10}. (Select the closest number.)

A.) 1.95295 **B.)** 1.95323

C.) 1.95326 **D.)** 1.95331

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 4).*

Problem 5. Use (compound) Simpson's rule with 4 subintervals to estimate π by approximating the integral

$$\int_0^1 \frac{4}{x^2+1}\,\mathrm{d}x.$$

A.) $\frac{102573}{32650}$ **B.)** $\frac{5323}{1275}$

C.) $\frac{355}{113}$ **D.)** $\frac{8011}{2550}$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 5).*

Problem 6. Given the data points $(0, 1), (1, 3), (2, 3), (3, 4), (4, 5), (5, 7)$ find the leastsquares fit of the form

$$y = \frac{1}{5}x^2 + b.$$

(Note you have only to find the one parameter b.)

A.) $b = 0$ **B.)** $b = 1$

C.) $b = 2$ **D.)** $b = 3$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 6).*

Problem 7. Let

$$s(x) = \begin{cases} -ax^3 + \frac{15}{4}x + 2, & \text{for } 0 \leq x \leq 1 \\ \frac{3}{4}x^3 - bx^2 + \frac{33}{4}x + \frac{1}{2} & \text{for } 1 \leq x \leq 2 \end{cases}$$

Choose a and b such that $s(x)$ is a natural cubic spline. Then

A.) $a = \frac{9}{2}$ **B.)** $a = \frac{3}{4}$

C.) $a = \frac{1}{2}$ **D.)** $a = \frac{7}{4}$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 7).*

Problem 8. Determine a, b and c such that the (silly) quadrature formula

$$Q(f) = a\, f(0) + b\, f\left(\frac{5}{8}\right) + c\, f(1)$$

is exact for the integral

$$\int_0^1 f(x)\, x^2 \, \mathrm{d}x$$

if $f(x)$ is a polynomial of degree ≤ 2, that is, is exact for $f(x) = 1$, $f(x) = x$ and $f(x) = x^2$. Then a is -

A.) $a = \frac{16}{76}$ **B.)** $a = \frac{7}{60}$

C.) $a = \frac{1}{300}$ **D.)** $a = \frac{2}{9}$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 8).*

Use the backs of the test pages for scratch work.