Mth 351 Test 2 | Name: ID:

Bent Petersen `351summer2001_test_2.tex` August 15, 2001 Time: 50 minutes.

Instructions: $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to write your name in the space above.

- *You may use one note-sheet prepared in advance. You must put your name on your note-sheet, but do not turn in your note-sheet. Your note-sheet must be letter size, 8.5×11 inches, or A4 paper, 21×29.7 cm, or smaller. You may write on both sides of your note-sheet.*
- *Note-sheets may not be shared. If you do not bring a note-sheet you will have to do without any help notes.*
- *You may not use any books, notebooks nor additional note-sheets.*
- *You may use a calculator. Calculators and other equipment may not be shared.*
- *For work-out problems sketch your work neatly. Highlight your answer by drawing a frame around it. Scratch out irrelevant or incorrect work so it will be clear what you are submitting as a solution. Give exact answers when possible. Simplify your answer when reasonable to do so. Partial credit will be assigned only for relevant, clear, correct, legible work. If you do not show some relevant work or explain your solution, your grade may be 0.*
- *For multiple-choice problems indicate your choice in the answer box provided. You need not show any work nor offer any explanations for your answer. If you need to do some work, you may do it in the space provided, if any, or on the back of the examination sheets, but your work will not be graded.* ***You will be graded only on the letter you select and put in the provided answer box.*** *Note this test does not use a scantron.*
- *Use the backs of the examination sheets for scratch work.*

Please note $\log(x)$ means the natural logarithm of x on this test.

Problem 1. *(25 points if correct, 0 points if wrong).* Use the bisection method to estimate the root of the polynomial $p(x) = 3\,x^4 - 14\,x^3 + 24\,x^2 - 4\,x - 3$ in the interval $[0, 1]$ with an error of at most $\frac{1}{8}$.

A.) $\frac{1}{2}$ **B.)** $\frac{5}{8}$
C.) $\frac{3}{4}$ **D.)** $\frac{7}{8}$ **E.)** None of the above.

☐ *←Letter corresponding to your answer to problem 1.*

Problem 2. *(25 points if correct, 0 points if wrong).* Find the interpolation polynomial $p(x)$ of degree ≤ 2 for the function $f(x) = \cos(x)$ with nodes at $x = 0$, $\frac{\pi}{2}$, and π. Then compute $p(\frac{\pi}{4})$.

A.) $\frac{1}{2}$ **B.)** $\frac{\sqrt{2}}{2}$
C.) 0 **D.)** $1 - \frac{1}{2\pi}$ **E.)** None of the above.

☐ *←Letter corresponding to your answer to problem 2.*

Problem 3. *(25 points if correct, 0 points if wrong).* The $3-$point Gauss quadrature method normalized for the interval $[0, 4]$ is given by

$$G_3(f) = \frac{10}{9}\, f\left(2 - \frac{2}{5}\sqrt{15}\right) + \frac{16}{9}\, f(2) + \frac{10}{9}\, f\left(2 + \frac{2}{5}\sqrt{15}\right).$$

Without doing too much calculation compute $G_3\left(x^5\right)$.

A.) 0 **B.)** $\frac{2048}{3}$
C.) $\frac{1}{6}$ **D.)** $\frac{512\sqrt{15}}{3}$ **E.)** None of the above.

☐ *←Letter corresponding to your answer to problem 3.*

Problem 4. *(25 points if correct, 0 points if wrong).* Let f be a smooth function. Use Taylor polynomials in h to compute

$$\lim_{h\to 0} \frac{3\,f(a-h) - 4\,f(a) + f(a+3h)}{h^2}.$$

A.) $f'(a)$ **B.)** $3\,f'(a) + 3\,f''(a)$
C.) $f''(a)$ **D.)** $6\,f''(a)$ **E.)** None of the above.

☐ ←*Letter corresponding to your answer to problem 4.*

Problem 5. *(25 points if correct, 0 points if wrong).* Denote by T_n the compound trapezoidal rule with n subintervals (for some function f on some interval $[a, b]$). If n is even we can apply Richardson extrapolation to obtain Simpson's rule $S_n = \frac{4}{3}T_n - \frac{1}{3}T_{n/2}$. If n is divisible by 4 we can extrapolate once again to obtain Bode's rule $B_n = \frac{16}{15}S_n - \frac{1}{15}S_{n/2}$. Given

$$T_1 = 1.1063582750, \quad T_2 = 0.8005457781, \quad T_3 = 0.7517873921, \quad T_4 = 0.7354090855, \quad T_5 = 0.7279356090, \cdots$$

compute B_4. Choose the number below closest to B_4.

A.) 0.6986082790 **B.)** 0.7136968546
C.) 0.7147027596 **D.)** 0.7148071457 **E.)** 0.7279356090

☐ ←*Letter corresponding to your answer to problem 5.*

Problem 6. *(25 points if correct, 0 points if wrong).* Recall the Čebyšev nodes for degree n interpolation on $[-1, 1]$ are given by

$$x_k = -\cos\left(\frac{2k+1}{2n+1}\,\pi\right), \quad k = 0, \cdots, n.$$

In particular for $n = 4$ we have (approximately)

$$\text{nodes} = [-.95105, -.58778, 0, .58778, .95105]$$

If $p(x)$ is the interpolation polynomial of degree 4 for e^x with the above Čebyšev nodes then

$$p(x) = 0.043315\,x^4 + 0.17729\,x^3 + 0.49970\,x^2 + 0.99739\,x + 1.000.$$

Use this result to estimate $\sqrt{\mathrm{e}}$. Choose the closest answer from those listed below.

A.) 2.7177 **B.)** 1.6490
C.) 1.6487 **D.)** 1.6485 **E.)** 1.6453

☐ ←*Letter corresponding to your answer to problem 6.*

Additional test policies for this class are provided on my web page `http://ucs.orst.edu/~peterseb`.

Use the space below for your scratch work.

Please do not write in the boxes to the right. They are for your grades.

Do not be concerned if there are more boxes than pages.

Letter Grade
☐ *This test only*
☐ *Cummulative*

Pg. 1	Pg. 2	Pg. 3	Pg. 4	Pg. 5	Pg. 6	Pg. 7	Pg. 8	Pg. 9	Pg. 10	Total

Note: There are 6 problems for a total of 150 points.