

Here is a list of common formulæ for numeric differentiation. I have not included any approximations of order one since they are not very useful. Be careful if you decide to use any of these formulæ – the formulæ with 6 or more terms in the numerator show significant roundoff errors even at moderate step sizes  $h$ .

$$\begin{aligned}
 * \quad f^{(1)}(a) &= \frac{-f(a-h) + f(a+h)}{2h} + \mathcal{O}(h^2) \\
 f^{(1)}(a) &= \frac{-3f(a) + 4f(a+h) - f(a+2h)}{2h} + \mathcal{O}(h^2) \\
 f^{(1)}(a) &= \frac{-2f(a-h) - 3f(a) + 6f(a+h) - f(a+2h)}{6h} + \mathcal{O}(h^3) \\
 f^{(1)}(a) &= \frac{-11f(a) + 18f(a+h) - 9f(a+2h) + 2f(a+3h)}{6h} + \mathcal{O}(h^3) \\
 * \quad f^{(1)}(a) &= \frac{f(a-2h) - 8f(a-h) + 8f(a+h) - f(a+2h)}{12h} + \mathcal{O}(h^4) \\
 f^{(1)}(a) &= \frac{-3f(a-h) - 10f(a) + 18f(a+h) - 6f(a+2h) + f(a+3h)}{12h} + \mathcal{O}(h^4) \\
 f^{(1)}(a) &= \frac{-25f(a) + 48f(a+h) - 36f(a+2h) + 16f(a+3h) - 3f(a+4h)}{12h} + \mathcal{O}(h^4) \\
 \\
 * \quad f^{(2)}(a) &= \frac{f(a-h) - 2f(a) + f(a+h)}{h^2} + \mathcal{O}(h^2) \\
 f^{(2)}(a) &= \frac{2f(a) - 5f(a+h) + 4f(a+2h) - f(a+3h)}{h^2} + \mathcal{O}(h^2) \\
 * \quad f^{(2)}(a) &= \frac{-f(a-2h) + 16f(a-h) - 30f(a) + 16f(a+h) - f(a+2h)}{12h^2} + \mathcal{O}(h^4) \\
 f^{(2)}(a) &= \frac{11f(a-h) - 20f(a) + 6f(a+h) + 4f(a+2h) - f(a+3h)}{12h^2} + \mathcal{O}(h^3) \\
 f^{(2)}(a) &= \frac{35f(a) - 104f(a+h) + 114f(a+2h) - 56f(a+3h) + 11f(a+4h)}{12h^2} + \mathcal{O}(h^3) \\
 \\
 * \quad f^{(3)}(a) &= \frac{-f(a-2h) + 2f(a-h) - 2f(a+h) + f(a+2h)}{2h^3} + \mathcal{O}(h^2) \\
 f^{(3)}(a) &= \frac{-3f(a-h) + 10f(a) - 12f(a+h) + 6f(a+2h) - f(a+3h)}{2h^3} + \mathcal{O}(h^2) \\
 f^{(3)}(a) &= \frac{-5f(a) + 18f(a+h) - 24f(a+2h) + 14f(a+3h) - 3f(a+4h)}{2h^3} + \mathcal{O}(h^2) \\
 \\
 * \quad f^{(4)}(a) &= \frac{f(a-2h) - 4f(a-h) + 6f(a) - 4f(a+h) + f(a+2h)}{h^4} + \mathcal{O}(h^2)
 \end{aligned}$$

In general the expected order of the estimate is the number of points minus the order of the derivative. In the central (\*) formulæ for even order derivatives, the order of the estimate is one higher than expected. In the central (\*) formulæ for odd order derivatives, function evaluation at the central point drops out.

Note in each non-central formula we can interchange the number of forward and backward steps by replacing  $h$  by  $-h$ . These non-central formulæ are useful near the ends of a table of data, where there are not enough data points for a central (\*) formula.