Mth 351 Sample Problems | Name: ID:

Bent Petersen `351u2002_samples_2.tex` Aug 10, 2002 Time: xx minutes.

Instructions: $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to write your name in the space above.

- *You may use one note-sheet prepared in advance. You must put your name on your note-sheet, but do not turn in your note-sheet. Your note-sheet must be letter size, 8.5×11 inches, or A4 paper, 21×29.7 cm, or smaller. You may write on both sides of your note-sheet.*
- *Note-sheets may not be shared. If you do not bring a note-sheet you will have to do without any help notes.*
- *You may not use any books, notebooks nor additional note-sheets.*
- *You may use a calculator. Calculators and other equipment may not be shared.*
- *For work-out problems sketch your work neatly. Highlight your answer by drawing a frame around it. Scratch out irrelevant or incorrect work so it will be clear what you are submitting as a solution. Give exact answers when possible. Simplify your answer when reasonable to do so. Partial credit will be assigned only for relevant, clear, correct, legible work. If you do not show some relevant work or explain your solution, your grade may be 0.*
- *For multiple-choice problems indicate your choice in the answer box provided. You need not show any work nor offer any explanations for your answer. If you need to do some work, you may do it in the space provided, if any, or on the back of the examination sheets, but your work will not be graded.* ***You will be graded only on the letter you select and put in the provided answer box.*** *Note this test does not use a scantron.*
- *Use the backs of the examination sheets for scratch work.*

Please note $\log(x)$ means the natural logarithm of x on this test.

Problem 1. Find the interpolation polynomial of degree ≤ 2 through the points $(1,1)$, $(2,0)$ and $(3,-3)$.

Ans: $-x^2 + 2x$

Problem 2. Find the natural cubic spline with knots $(0,1)$, $(2,0)$ and $(3,1)$.

Ans: $1 - x + \frac{1}{8}x^3$, $4 - \frac{11}{2}x = \frac{9}{4}x^2 - \frac{1}{4}x^3$

Problem 3. Let T_n be the compound trapezoidal rule with n subintervals (so $n+1$ nodes). Let S_n be the compound Simpson's rule with n subintervals (n even). Suppose for a certain function f on an interval $[a,b]$ we have $T_3(f) = 8.812138$, $T_6(f) = 5.893109$ and $T_{12}(f) = 5.109739$. Compute $S_6(f)$ and $S_{12}(f)$.

Ans: 4.920099, 4.848616

Problem 4. Use T_2 to estimate $\int_1^3 x^4 \, \mathrm{d}x$.

Ans: 57

Problem 5. Use S_2 to estimate $\int_2^4 x^5 \, \mathrm{d}x$.

Ans: 676

Problem 6. Find the cubic Hermite interpolation polynomial $Q(x)$ with $Q(1) = 0$, $Q(2) = 2$, $Q'(1) = -2$ and $Q'(2) = -1$.

Ans: $20 - 45x + 32x^2 - 7x^3$

Problem 7. Find the line of least squares best fit for the points $(0,-1)$, $(1,2)$, $(2,1)$ and $(4,-1)$.

Ans: $y = -\frac{1}{5}x + \frac{3}{5}$

Problem 8. Find the curve of the form $y = ax^2 + bx$ which is the least squares best fit for the data points $(0, 0)$, $(1, 1)$, $(2, 2)$ and $(3, 4)$.

Ans: $y = \frac{9}{38}x^2 + \frac{23}{38}x$

Problem 9. According to Kepler's third law the square of the period of a planet is proportional to the cube of its mean distance (semi-major axis) from the sun. Here is some data for our solar system:

Planet	semi-major axis (s) 10^6 kilometers	orbital period (p) years
Mercury	57.9	0.241
Venus	108.1	0.615
Earth	149.5	1.000
Mars	227.8	1.881
Jupiter	778.0	11.860
Saturn	1426.0	29.460
Uranus	2868.0	84.010
Neptune	4494.0	164.800
Pluto	5896.0	247.700

Find the least squares best fit of the form $p = C\, s^{3/2}$ to the data above.

Ans: $p = .0005470869275 s^{3/2}$

Problem 10. Gauss quadrature on $[-1, 1]$ with three knots is given by

$$G_3(f) = \frac{5}{9}\, f\left(\frac{-\sqrt{15}}{5}\right) + \frac{8}{9}\, f(0) + \frac{5}{9}\, f\left(\frac{\sqrt{15}}{5}\right).$$

Use G_3 to estimate $\int_{-1}^{1} x^6 + 9,793,238,462,643,383,279,502,884,197\, x^5 \ \mathrm{d}x$

Ans: $\frac{6}{25}$
