Mth 351 Test 1 | Name: ID:

Bent Petersen `351u2002_test_1.tex` July 22, 2001 Time: 50 minutes.

Instructions: $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to write your name in the space above.

- *You may use one note-sheet prepared in advance. You must put your name on your note-sheet, but do not turn in your note-sheet. Your note-sheet must be letter size, 8.5×11 inches, or A4 paper, 21×29.7 cm, or smaller. You may write on both sides of your note-sheet.*
- *Note-sheets may not be shared. If you do not bring a note-sheet you will have to do without any help notes.*
- *You may not use any books, notebooks nor additional note-sheets.*
- *You may use a calculator. Calculators and other equipment may not be shared.*
- *For work-out problems sketch your work neatly. Highlight your answer by drawing a frame around it. Scratch out irrelevant or incorrect work so it will be clear what you are submitting as a solution. Give exact answers when possible. Simplify your answer when reasonable to do so. Partial credit will be assigned only for relevant, clear, correct, legible work. If you do not show some relevant work or explain your solution, your grade may be 0.*
- *For multiple-choice problems indicate your choice in the answer box provided. You need not show any work nor offer any explanations for your answer. If you need to do some work, you may do it in the space provided, if any, or on the back of the examination sheets, but your work will not be graded.* ***You will be graded only on the letter you select and put in the provided answer box.*** *Note this test does not use a scantron.*
- *Use the backs of the examination sheets for scratch work.*

Please note $\log(x)$ means the natural logarithm of x on this test.

Problem 1. *(20 points if correct, 0 points if wrong).* Let the function f be defined by $f(x) = x + \cos(x) - 2.1$. Which of the following intervals contains a root of f?

A.) $[0, 1]$ **B.)** $[1, 2]$
C.) $[2, 3]$ **D.)** $[3, 4]$ **E.)** $[4, 5]$

[ ] ←*Letter corresponding to your answer to problem 1.*

Problem 2. *(20 points).* Let $g(x) = 1 + \frac{1}{x}$. **Part (A)** Find a positive fixed point for g. **Part (B)** Let $x_0 = \frac{3}{2}$. Compute the first three "fixed point iterates" x_1, x_2 and x_3. **Part (C)** Do the iterates appear to be converging to the fixed point found in part (A)?

Problem 3. *(20 points if correct, T points if wrong).* he polynomial

$$p(x) = x^4 - x^3 - 7x^2 + x + 8$$

has a root in the interval $[1, 2]$. Suppose you take $x_0 = 1$ as an estimate of the root and apply Newton's method once to obtain a new estimate x_1. From the list below, choose the number closest to x_1.

A.) 0.833 **B.)** 1.143
C.) 1.159 **D.)** 1.167 **E.)** 1.210

☐ ←*Letter corresponding to your answer to problem 3.*

Problem 4. *(20 points if correct, T points if wrong).* he polynomial

$$p(x) = x^4 - x^3 - 7x^2 + x + 8$$

has a root in the interval $[1, 2]$. Suppose you take $x_0 = 1$ and $x_1 = 2$ as initial estimates of the root and apply the secant method once to obtain a new estimate x_2. From the list below, choose the number closest to x_2.

A.) 0.833 **B.)** 1.143
C.) 1.159 **D.)** 1.167 **E.)** 1.210

☐ ←*Letter corresponding to your answer to problem 4.*

Problem 5. *(20 points if correct, 0 points if wrong).* Let S be a set of 9 vectors in $\mathbb{R}^8$. Which of the following statements is (always) true?

A.) S is linearly independent **B.)** S spans $\mathbb{R}^8$
C.) S is linearly dependent **D.)** S does not span $\mathbb{R}^8$ **E.)** None of the above.

☐ ←*Letter corresponding to your answer to problem 5.*

Problem 6. *(20 points if correct, 0 points if wrong).* Let S be a set of 7 vectors in $\mathbb{R}^8$. Which of the following statements is (always) true?

A.) S is linearly independent **B.)** S spans $\mathbb{R}^8$
C.) S is linearly dependent **D.)** S does not span $\mathbb{R}^8$ **E.)** None of the above.

☐ ←*Letter corresponding to your answer to problem 6.*

Problem 7. *(20 points).* The 3×3 matrix A has an LU factorization where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

If $b = \begin{bmatrix} 1 & 2 & 2 \end{bmatrix}^T$ solve $Ax = b$.

Problem 8. *(20 points).* The equations

$$\begin{aligned} y^2 - xy + x^2 - 3 &= 0 \\ y^2 + 2xy + x^2 + x - 2 &= 0 \end{aligned}$$

have four simultaneous solutions (roots). Graphically we determine one of the solutions lies near the point $(1, -2)$. Use Newton's method with just one iteration and initial guess $x_0 = 1$ and $y_0 = -2$ to obtain an approximation of one of the solutions.
