Mth 351 Test 2 | Name: ID:

Bent Petersen `351u2002_test_2.tex` Aug 14, 2001 Time: 50 minutes.

Instructions: $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to write your name in the space above.

- *You may use one note-sheet prepared in advance. You must put your name on your note-sheet, but do not turn in your note-sheet. Your note-sheet must be letter size, 8.5×11 inches, or A4 paper, 21×29.7 cm, or smaller. You may write on both sides of your note-sheet.*
- *Note-sheets may not be shared. If you do not bring a note-sheet you will have to do without any help notes.*
- *You may not use any books, notebooks nor additional note-sheets.*
- *You may use a calculator. Calculators and other equipment may not be shared.*
- *For work-out problems sketch your work neatly. Highlight your answer by drawing a frame around it. Scratch out irrelevant or incorrect work so it will be clear what you are submitting as a solution. Give exact answers when possible. Simplify your answer when reasonable to do so. Partial credit will be assigned only for relevant, clear, correct, legible work. If you do not show some relevant work or explain your solution, your grade may be 0.*
- *For multiple-choice problems indicate your choice in the answer box provided. You need not show any work nor offer any explanations for your answer. If you need to do some work, you may do it in the space provided, if any, or on the back of the examination sheets, but your work will not be graded.* ***You will be graded only on the letter you select and put in the provided answer box.*** *Note this test does not use a scantron.*
- *Use the backs of the examination sheets for scratch work.*

Please note $\log(x)$ *means the natural logarithm of* x *on this test. Note also that you will need a simple calculator for some of the problems.*

Problem 1. *(20 points if correct, 0 points if wrong).* If we use the compound trapezoidal rule with 4 intervals to estimate the integral $\int_0^2 x^4 \, \mathrm{d}x$ we obtain (select the closest number):

A.) 6.4000 **B.)** 6.4167
C.) 6.9733 **D.)** 7.0625 **E.)** 7.3125

☐ *←Letter corresponding to your answer to problem 1.*

Problem 2. *(20 points if correct, 0 points if wrong).* If we use the compound Simpson's rule with 4 intervals to estimate the integral $\int_0^2 x^4 \, \mathrm{d}x$ we obtain (select the closest number):

A.) 6.4000 **B.)** 6.4167
C.) 6.9733 **D.)** 7.0625 **E.)** 7.3125

☐ *←Letter corresponding to your answer to problem 2.*

Problem 3. *(20 points).* Find the cubic Hermite interpolation polynomial $Q(x)$ with $Q(0) = 1$, $Q(2) = 1$, $Q'(0) = 1$ and $Q'(2) = 1$.

Problem 4. *(20 points).* Find the interpolation polynomial of degree at most 4 through the points $(1, 0)$, $(2, 0)$, $(3, 0)$, $(4, 6)$ and $(5, -8)$. (Note, a bit of thought may save a lot of computation.)

Problem 5. *(20 points).* Suppose we want a quadrature rule (approximate integration rule) for functions defined on the interval $[0, 1]$. Our quadrature rule should be of the form

$$H(f) = a\, f(0) + b\, f(1/4) + c\, f(3/4) + d\, f(1)$$

and should be exact for polynomials of degree 3 or less. Compute the coefficients a, b, c, d.

Problem 6. *(20 points).* Find the natural cubic spline with knots $(-1, 2)$, $(0, 0)$, $(1, 2)$.

Problem 7. *(20 points).* In 1202 Leonardo of Pisa, also known as Fibonacci, in his book, *Liber Abaci*, introduced Arabic, Greek and Hindu arithmetic, geometry and algebra into Europe, and introduced the Hindu-Arabic place-valued decimal system and the use of Arabic numerals into Europe. Around 1225, in his book *Flos* he studies the equation $x^3 + 2x^2 + 10x - 20 = 0$ (introduced earlier by Omar Khayyam). Fibonacci points out the equation has one real root and that it is not possible to find the root by any classical methods. He then explicitly says he worked to find an *approximate solution* – the beginnings of our subject. He obtains the approximation 1.3688081075 which is pretty accurate. We have no idea how he did it. It is possible he re-arranged the equation as

$$x = 20\,\frac{1}{x^2 + 2x + 10}$$

and then used what we would call a fixed-point iteration. Try it! Start with an initial guess of 1.3 and iterate 3 times. What do you get?

Note: The root is about $1.368808107821372635227414330 0\cdots$. Fibonacci was very close indeed!

Problem 8. *(20 points).* Write a brief *technical* essay describing what you enjoyed most in Mth 351. Your essay should contain at least one equation or *mathematical* expression and should be written in reasonably good English.
