Mth 351 Assignment 2 | Summer 2005 | Name:

Bent Petersen `351u2005-002.tex` Due date: Wed July 6, 2005

Instructions: Please supply your solution(s) by the due date in the space provided below. Continue on to the back of the sheet if you need more space. If you turn in additional sheets please staple them in order to the back of this sheet and put your name on each sheet. For additonal comments and instructions check my webpage http://oregonstate.edu/~peterseb

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Problem 2.1 Imagine an (impractical) binary floating point representation with exponent ranging from -63 to 64 and with mantissa of length 12 bits. Assume only normalized representations are used, chopping is used, and there is no packing. Estimate the error made when storing $\pi = 11.00100100001111110110101010001000100010\cdots$, for example, find a close upper bound for the error. (Yes, 10^{18} is an upper bound for the error, but it is not very close!)

Problem 2.2 Imagine an (impractical) binary floating point representation with exponent ranging from -63 to 64 and with mantissa of length 12 bits. Assume only normalized representations are used, chopping is used, and there is no packing. Give a good estimate of the error made when storing 13.77.

A real number x_a is said to have at least m significant decimal digits as an approximation to the real number $x_t \neq 0$ provided that

$$\left| \frac{x_t - x_a}{x_t} \right| \leq 5 \times 10^{-m-1}.$$

The best value (largest) of m is called the *number of significant decimal digits in x_a as an approximation to x_t* or simply the *number of significant digits in x_a* when x_t and *“decimal”* are understood. The best value of m is clearly given by

$$m = \left\lfloor -\log_{10}\left(2\left|\frac{x_t - x_a}{x_t}\right|\right)\right\rfloor.$$

Here $\lfloor \quad \rfloor$ indicates the *greatest integer function* otherwise known as *floor*.

Problem 2.3 The fractions $\frac{22}{7}$, $\frac{333}{106}$ and $\frac{355}{113}$ are well-known approximations to π. Use the formula above to find the the number of significant decimal digits in each. Does your understanding of significant digits agree with your calculated result?