Mth 351 Assignment 3 | Summer 2005 | Name:

Bent Petersen `351u2005-003.tex` Due date: Wed July 13, 2005

Instructions: Please supply your solution(s) by the due date in the space provided below. Continue on to the back of the sheet if you need more space. If you turn in additional sheets please staple them in order to the back of this sheet and put your name on each sheet. For additonal comments and instructions check my webpage http://oregonstate.edu/˜peterseb

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Problem 3.1 While computers generally use binary floating point number representations, calculators often use decimal floating point representations, probably to avoid time consuming conversions to keep the display updated. It is not difficult to see

$$(2727273)(12674893) = 34567893456789 \text{ and } (3718196)(9296953) = 34567893456788$$

and therefore

$$(2727273)(12674893) - (3718196)(9296953) = 1.$$

What answers are you likely to get if you compute this number on a 10 digit, a 12 digit, and a 14 digit calculator?

Problem 3.2 There are many variations on Newton's method for approximating roots

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

For example, Halley's method is

$$x_{n+1} = x_n - \left(\frac{f'(x_n)}{f(x_n)} - \frac{f''(x_n)}{2f'(x_n)}\right)^{-1}$$

and Olver's method if

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{f''(x_n)f(x_n)^2}{2f'(x_n)^3}.$$

Apply all three methods to $f(x) = x^5 - 2x^2 + 2$ with $x_0 = -2.0$. Given that $f(x)$ has a simple root (you will not need all this precision) at

$$-0.86806873518420693013546552560477468665547539233980902748091599559168494157683 8\ldots$$

how well do the methods above fare? Note we expect Newton's method to converge quadratically, but what about Halley and Olver's methods? What rate of convergence do you appear to observe? Note 5 to 8 iterations will suffice to see what's happening. (Double precision on a computer ought to suffice to get meaningful results. You should even be able to get away with using a spreadsheet. A calculator is unlikely to do the job.)

Problem 3.3 The *Regula Falsi* (false position or false rule [but not in the sense of line]) method for approximating roots is very ancient. It is similar to the bisection method except we divide the interval according to the magnitude of the function at the endpoints (closer to the smaller magnitude, rather than in half). The division point is the same as the next estimate in the secant method. Thus if f is continuous on $[a, b]$ and $f(a)f(b) < 0$ define

$$c = b - \frac{f(b)(a-b)}{f(a) - f(b)} = \frac{bf(a) - af(b)}{f(a) - f(b)}.$$

Now if $f(a)f(c) < 0$ use the interval $[a, c]$, else use $[c, b]$, and proceed as before. At each stage the interval in hand is guaranteed to contain a root. Apply the *Regula Falsi* method to $x^2 - 2$ say with 5 iterations and starting with the interval $[1, 2]$. Use the final value of c as your approxiamtion to $\sqrt{2}$. What is the error? How does this compare with the result of the bisection method (binary search) with the same number of iterations?