Mth 351 Assignment 4 | Summer 2005 | Name:

Bent Petersen `351u2005-004.tex` Due date: Wed June 27, 2005

Instructions: Please supply your solution(s) by the due date in the space provided below. Continue on to the back of the sheet if you need more space. If you turn in additional sheets please staple them in order to the back of this sheet and put your name on each sheet. For additonal comments and instructions check my webpage http://oregonstate.edu/˜peterseb

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Problem 4.1 Let

$$f(x) = \frac{x+1}{10x^2+1}$$

and let $P_E(x)$ be the interpolation polynomial for $f(x)$ with nodes at

$$-1, -\frac{2}{3}, -\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}.1$$

Compute $P_E(x)$ and by plotting $f(x) - P_E(x)$ estimate the maximum absolute error in $P_E(x)$ as an approximation to $f(x)$ on the interval $[-1, 1]$.

Problem 4.2 Let $f(x)$ be defined as in the previous problem and let $P_S(x)$ be the interpolation polynomial for $f(x)$ with nodes at

$$-1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{12}, \frac{1}{2}, \frac{3}{4}.1$$

Compute $P_S(x)$ and by plotting $f(x) - P_S(x)$ estimate the maximum absolute error in $P_S(x)$ as an approximation to $f(x)$ on the interval $[-1, 1]$.

Problem 4.3 Let $f(x)$ be defined as in the previous problem and let $P_C(x)$ be the Čebyšev interpolation polynomial of degree at most 6 for $f(x)$, that is, with the Čebyšev nodes

$$-\cos\left(\frac{\pi}{14}\right), -\cos\left(\frac{3\pi}{14}\right), -\cos\left(\frac{5\pi}{14}\right), -\cos\left(\frac{7\pi}{14}\right), -\cos\left(\frac{9\pi}{14}\right), -\cos\left(\frac{11\pi}{14}\right), -\cos\left(\frac{13\pi}{14}\right)$$

that is,

$$-\cos\left(\frac{\pi}{14}\right), -\cos\left(\frac{3\pi}{14}\right), -\cos\left(\frac{5\pi}{14}\right), 0, \cos\left(\frac{5\pi}{14}\right), \cos\left(\frac{3\pi}{14}\right), \cos\left(\frac{\pi}{14}\right)$$

Compute $P_C(x)$ and by plotting $f(x) - P_C(x)$ estimate the maximum absolute error in $P_C(x)$ as an approximation to $f(x)$ on the interval $[-1, 1]$.

Problem 4.4 Compare the errors in the previous 3 problems and write some comments. For example, it may be your work will show that $P_S(x)$ has a slightly smaller maximum absolute error than the Čebyšev interpolation polynomial $P_C(x)$. How can that be? What, exactly, is it that is minimized by choosing the Čebyšev nodes for interpolation?