

Bent Petersen 351u2005-test2.tex Test date: August 10 2005 Time: 70 minutes

**Instructions:**  $\implies$ 

If you do not read the instructions, then how will you know what to do? Read them now.

- This test is a multiple-choice test. Be sure you put your name on the scantron.
- You must mark your answer on the provided scantron. Fill in the appropriate bubbles on the scantron very carefully.
- You may use one 8.5 × 11 inch note sheet prepared in advance. You may write on both sides of your note sheet.
- Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes.
- You may not use any books, notebooks, additional note sheets nor note cards.
- You are expected to have a simple scientific calculator or a modest graphics calculator available for use on this test. Calculators and other equipment may not be shared.
- You may use a simple graphics calculator but not a laptop computer nor any device capable of extensive symbolic manipulation (other than your own brain).
- There are 9 multiple-choice problems worth 12 points each.

Be sure to enter all required information on the scantron.

Section Number: 001

Form Number: 001

**Important Notes:**

- Note that  $\log(x)$  means the *natural logarithm* of  $x$ , sometimes denoted by  $\ln(x)$ . The logarithm with base 10 will be denoted by  $\log_{10}(x)$ , the logarithm with base 2 will be denoted by  $\log_2(x)$ , and so on.
- Return only the scantron. You may keep the test (and your note sheet).
- Make certain your calculator is set to radian mode.

**Problem 1.** Let the function  $g$  be defined by  $g(x) = 1 + x - \frac{1}{4}x^2$ . Find a positive fixed point of  $g$ .

- A.) 1                      B.) 2  
C.)  $2 + 2\sqrt{2}$         D.)  $g$  has no positive fixed points    E.) None of the foregoing.

$\leftarrow$  Write letter corresponding to your answer here and mark it on the scantron (Problem 1).

**Problem 2.** For a certain function  $f$  let  $T_n(f)$  be the estimate of  $\int_0^1 f(x) dx$  obtained by using the (compound) trapezoidal rule with  $n$  subintervals. Let  $S_n(f)$  be the estimate of  $\int_0^1 f(x) dx$  obtained by using the (compound) Simpson's rule with  $n$  subintervals ( $n$  even in this case). If  $T_4(f) = 1.436531563$  and  $T_8(f) = 1.418825196$  compute  $S_8(f)$ .

- A.) 1.412923074        B.) 1.427678380  
C.) 1.418825196        D.) 2.119384611        E.) None of the foregoing.

$\leftarrow$  Write letter corresponding to your answer here and mark it on the scantron (Problem 2).

**Problem 3.** Devise an approximate quadrature rule (probably of no practical value)

$$Q(f) = af(0) + bf(3/4) + cf(1) \text{ to estimate } \int_0^1 f(x) \, dx$$

such that

$$Q(p) = \int_0^1 p(x) \, dx$$

when  $p(x)$  is a polynomial of degree  $\leq 2$ . That is, our rule is exact for 1,  $x$  and  $x^2$ . Find the coefficient  $a$ .

- A.)**  $5/18$     **B.)**  $8/9$   
**C.)**  $1/3$     **D.)**  $-1/6$     **E.)** None of the foregoing.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 3).

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**Problem 4.** Let  $p(x) = ax^3 + bx^2 + cx + d$  be the interpolation polynomial of degree  $\leq 3$  for the points  $(0, 2)$ ,  $(1, 2)$ ,  $(2, 4)$ , and  $(3, 2)$ . Then

- A.)**  $p(x) = -2x^3 + 7x^2 - 5x + 2$     **B.)**  $p(x) = 3x^3 + 10x^2 - 7x + 2$   
**C.)**  $p(x) = -x^3 + 4x^2 - 3x + 2$     **D.)**  $p(x) = x^3 - 2x^2 + x + 2$     **E.)** None of the foregoing.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 4).

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**Problem 5.** For a certain  $2 \times 2$  matrix  $A$  we have

$$A \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \text{and} \quad A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

Find  $A$ . The last row of  $A$  is

- A.)**  $[1, -1]$     **B.)**  $[-1, 1]$   
**C.)**  $[0, 1]$     **D.)**  $[1, 2]$     **E.)** None of the foregoing.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 5).

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**Problem 6.** A certain  $3 \times 3$  matrix  $A$  has an LU decomposition with

$$L := \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ \frac{2}{3} & -\frac{1}{3} & 1 \end{bmatrix} \quad U := \begin{bmatrix} 3 & 2 & 5 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}.$$

Solve the system  $Ax = b$  where  $b = [3, 6, 1]^T$  (note  $T$  indicates the transpose).

- A.)**  $x = [23/6, -11/2, 1/2]^T$     **B.)**  $x = [3, 3, 0]^T$   
**C.)**  $x = [23/6, -28/3, -31/6]^T$     **D.)**  $x = [3, -3, 0]^T$     **E.)** None of the foregoing.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 6).

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**Problem 7.** The world population in 1950 was 2.51 billion, in 1960 it was 3.15 billion, in 1970 it was 3.65 billion and in 1980 it was 4.20 billion. Fit this data with a least squares line  $p = at + b$  (line of regression). Now use your model to “guess” the population for 1990. (It will be a lousy guess.)

- A.) 4.61 billion    B.) 4.77 billion  
C.) 5.08 billion    D.) 5.30 billion    E.) None of the foregoing.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 7).

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**Problem 8.** Consider the linear system

$$\begin{aligned}3x + y + z &= 1 \\2x + 5y + 2z &= 3 \\2y + 3z &= -2\end{aligned}$$

Starting with an initial “guess”  $[0, 0, 0]^T$  compute the second Gauss–Jacobi iterate.

- A.)  $[0.503, 0.790, -1.193]$     B.)  $[0.333, 0.467, -0.978]$   
C.)  $[0.333, 0.600, -0.667]$     D.)  $[0.356, 0.733, -1.067]$     E.) None of the foregoing.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 8).

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**Problem 9.** Consider the linear system

$$\begin{aligned}3x + y + z &= 1 \\2x + 5y + 2z &= 3 \\2y + 3z &= -2\end{aligned}$$

Starting with an initial “guess”  $[0, 0, 0]^T$  compute the second Gauss–Seidel iterate.

- A.)  $[0.503, 0.790, -1.193]$     B.)  $[0.333, 0.467, -0.978]$   
C.)  $[0.333, 0.600, -0.667]$     D.)  $[0.356, 0.733, -1.067]$     E.) None of the foregoing.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 9).

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Use the backs of the test pages for scratch work.

*Enjoy the rest of your Summer!*