

Instructions: \implies

If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to write your name in the space above.

- You may use one note-sheet prepared in advance. You must put your name on your note-sheet, but do not turn in your note-sheet. Your note-sheet must be letter size, 8.5×11 inches, or A4 paper, 21×29.7 cm, or smaller. You may write on both sides of your note-sheet.
- Note-sheets may not be shared. If you do not bring a note-sheet you will have to do without any help notes.
- You may not use any books, notebooks nor additional note-sheets.
- You may use a calculator. Calculators and other equipment may not be shared.
- For work-out problems sketch your work neatly. Highlight your answer by drawing a frame around it. Scratch out irrelevant or incorrect work so it will be clear what you are submitting as a solution. Give exact answers when possible. Simplify your answer when reasonable to do so. Partial credit will be assigned only for relevant, clear, correct, legible work. If you do not show some relevant work or explain your solution, your grade may be 0.
- For multiple-choice problems indicate your choice in the answer box provided. You need not show any work nor offer any explanations for your answer. If you need to do some work, you may do it in the space provided, if any, or on the back of the examination sheets, but your work will not be graded. **You will be graded only on the letter you select and put in the provided answer box.** Note this test does not use a scantron.
- Use the backs of the examination sheets for scratch work.

You are expected to have a simple scientific calculator available for use on this test.

Problem 1. (20 points if correct, 0 points if wrong). The function

$$f(x) = 1.07 + 2.09x + \cos(0.99x)$$

has a root in the interval $[-1, 0]$. Choose the number below which is closest to the estimate of the root provided by using Newton's method once with initial guess $x_0 = -1$.

- A.)** -0.73325 **B.)** -0.83578
C.) -0.83846 **D.)** -1.16154 **E.)** -1.43973

←Letter corresponding to your answer to problem 1.

Problem 2. (20 points if correct, 0 points if wrong). The function

$$f(x) = 1.07 + 2.09x + \cos(0.99x)$$

has a root in the interval $[-1, 0]$. Choose the number below which is closest to the estimate of the root provided by using the secant method once with initial guess $x_0 = -1$ and $x_1 = 0$.

- A.)** -0.81454 **B.)** -0.83554
C.) -0.83589 **D.)** -0.84479 **E.)** -0.82354

←Letter corresponding to your answer to problem 2.

Problem 3. (20 points). Find a (*exact*) fixed point for the function

$$g(x) = \frac{x^2 + x + 15}{x^2}.$$

Problem 4. (20 points). Find the completely row reduced row echelon Gauss-Jordan canonical form of the 3×5 matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 6 \\ 3 & 4 & 5 & 7 & 9 \end{bmatrix}.$$

Problem 5. (20 points). For a certain symmetric positive definite 3×3 matrix A the Cholesky factorization is $A = C C^T$ where

$$C = \begin{bmatrix} \sqrt{6} & 0 & 0 \\ \frac{4}{3}\sqrt{6} & \frac{5}{3}\sqrt{3} & 0 \\ -\frac{2}{3}\sqrt{6} & \frac{2}{3}\sqrt{3} & X \end{bmatrix}.$$

If the $(3, 3)^{th}$ entry in A is 12 find X .

Problem 6. (20 points). Find the least squares fit of the form

$$y = a + bx^2$$

for the data

$$x = [0.0, 0.5, 1.0]$$

$$y = [1.72, 0.49, 2.30]$$

Problem 7. (20 points). A 3×3 matrix A has an LU-decomposition $A = LU$ where

$$U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -9 & -15 \\ 0 & 0 & -6 \end{bmatrix}.$$

If $Ly = b$ where $b = [1, 2, -1]^T$ has the solution $y = [1, -5, -5]^T$, then find the solution to $Ax = b$. (Here B^T indicates the transpose of B .)

Problem 8. (20 points if correct, 0 points if wrong). Let A be an $m \times n$ matrix and let $M = [A, b]$ be the augmented matrix of the system of linear equations $Ax = b$. Then $Ax = b$ has at least one solution if and only if

- A.)** $\text{rank}(A) < \text{rank}(M)$ **B.)** $\text{rank}(A) = \text{rank}(M)$
C.) $\text{rank}(A) > \text{rank}(M)$ **D.)** $\text{rank}(A) \neq \text{rank}(M)$ **E.)** None of the above.

←Letter corresponding to your answer to problem 8.
