

Graph Theory in Maple

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Maple 6

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Assignment 8 - due "never" (There is an OSU tradition of no new assignments during dead week.)

This worksheet contains a few comments on some of Maple's graph support. It was composed fairly quickly and is unlikely to be free of errors. If you find a serious error be sure to let me know.

```
[ > restart;
```

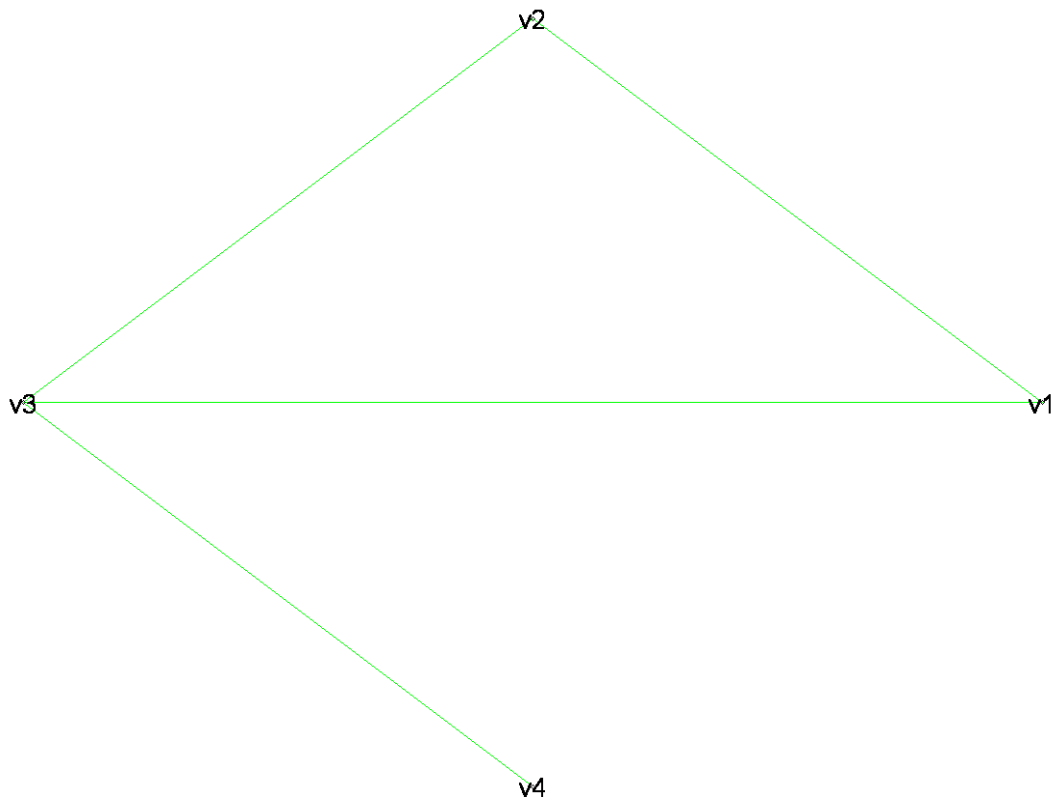
```
[ > with(networks):
```

Graphs are created by the `graph()` command. If V is the set of vertices and E is a set of edges, each described as a set of two vertices (undirected edge) or as a list of two vertices (directed edges), then `graph(V,E)` constructs the corresponding graph.

```
[ > G1:=graph( {v1,v2,v3,v4},{ {v1,v2},{v2,v3},{v3,v4},{v3,v1} } ):
```

Once we have defined a graph we can draw it:

```
[ > draw(G1);
```



The adjacency matrix is given by the `adjacency()` command:

```
> A1:=adjacency(G1);
```

$$A1 := \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Graphs may also be specified by using the `addvertex()` and `addedge()` commands:

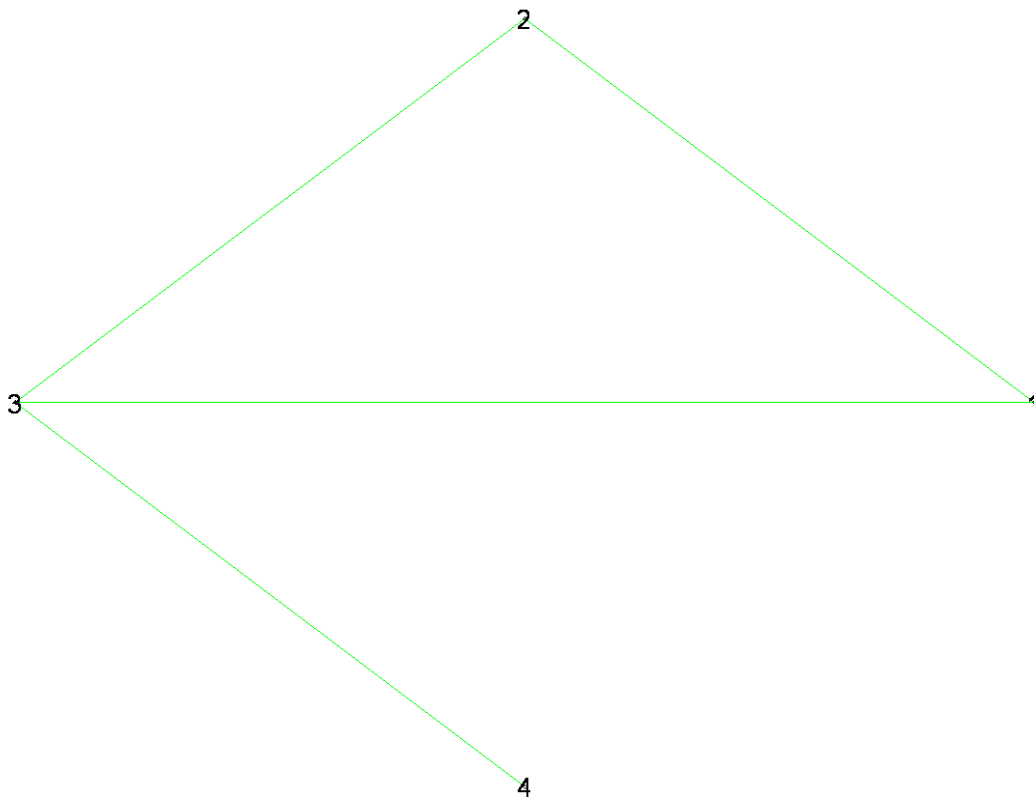
```
> new(G2):
```

```
> addvertex({1,2,3,4},G2):
```

```
> addedge(Cycle(1,2,3),G2):
```

```
> addedge({3,4},G2):
```

```
> draw(G2);
```



Instead of using `new()` and `addvertex()` as above we can use `void()` which creates a graph with no edges:

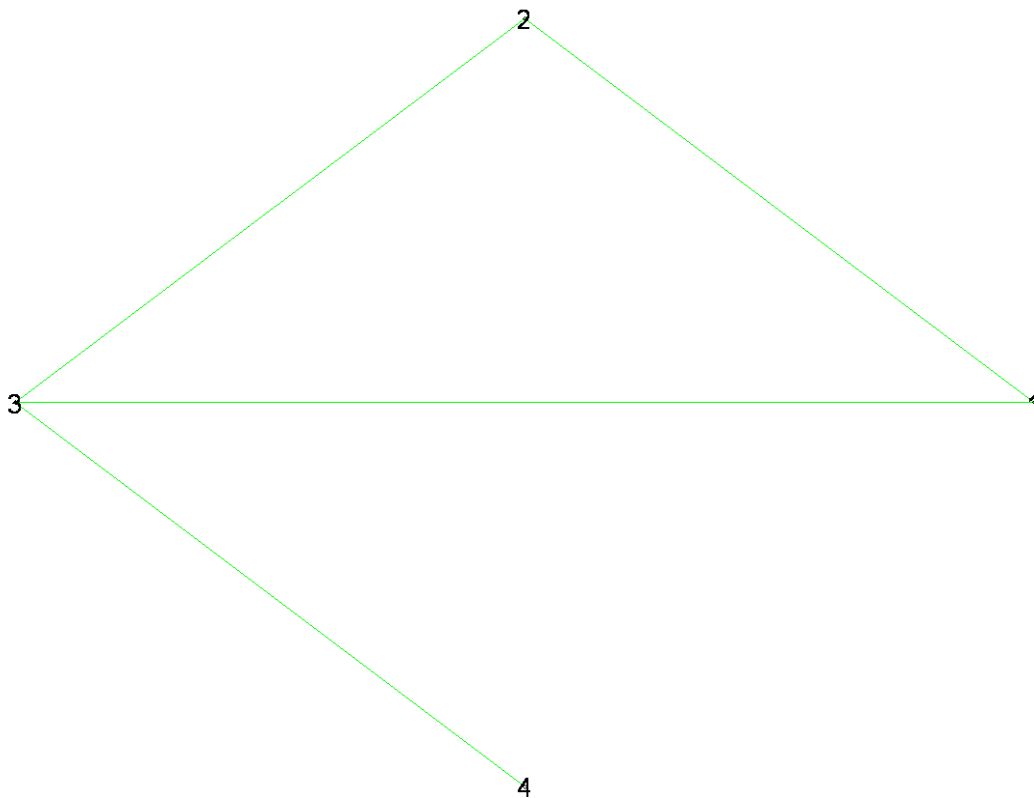
```

> G3:=void({1,2,3,4}):
> vertices(G3);
      {1,2,3,4}
> edges(G3);
      { }
  
```

We can now use the `addedge()` command to add any desired edges.

```

> addedge( {{1,2},{2,3},{3,4},{3,1}},G3);
      e1, e2, e3, e4
> draw(G3);
  
```



The characteristic polynomial of an undirected graph is just the characteristic polynomial of its adjacency matrix.

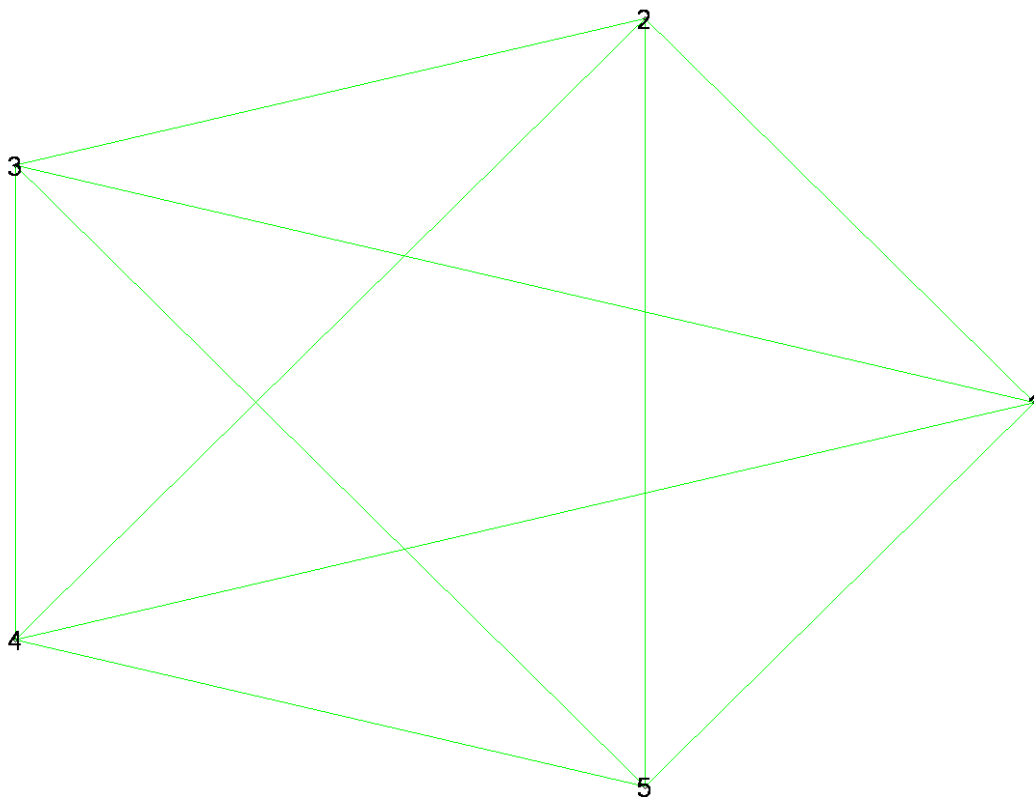
```
> p3:=sort(charpoly(G3,z),z);
```

$$p3 := z^4 - 4z^2 - 2z + 1$$

Maple has builtin definitions for complete graphs. Here is the complete graph K5

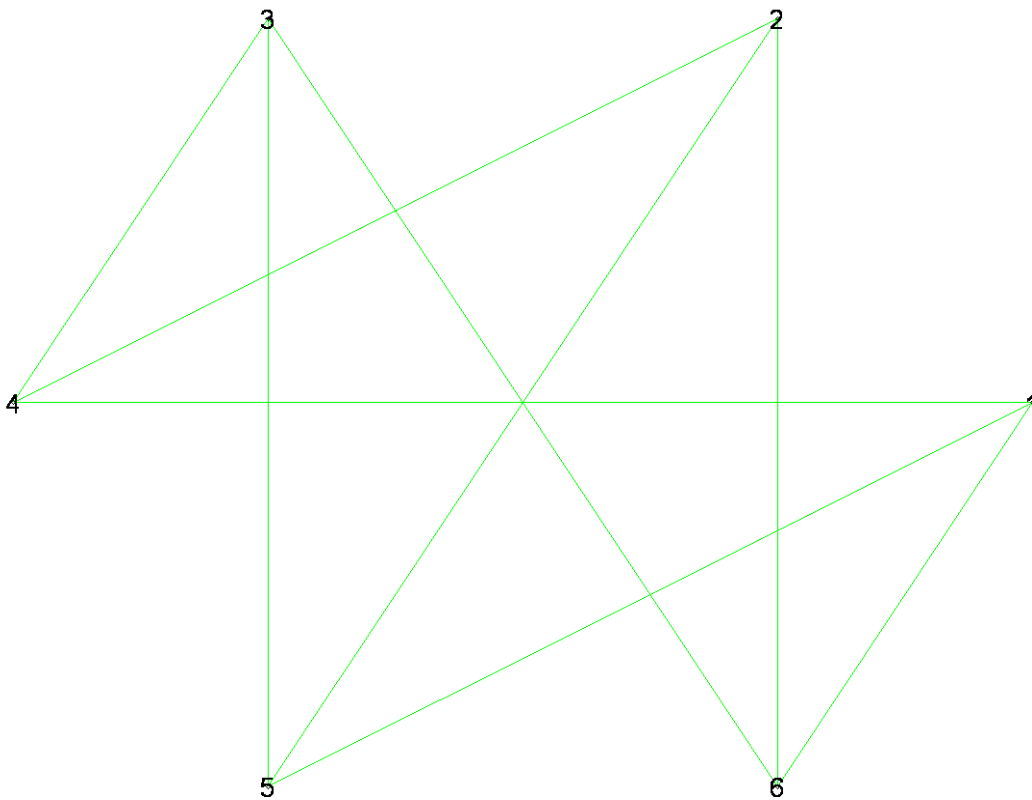
```
> K5:=complete(5):
```

```
> draw(K5);
```



As another example here is the complete bipartite graph $K_{3,3}$

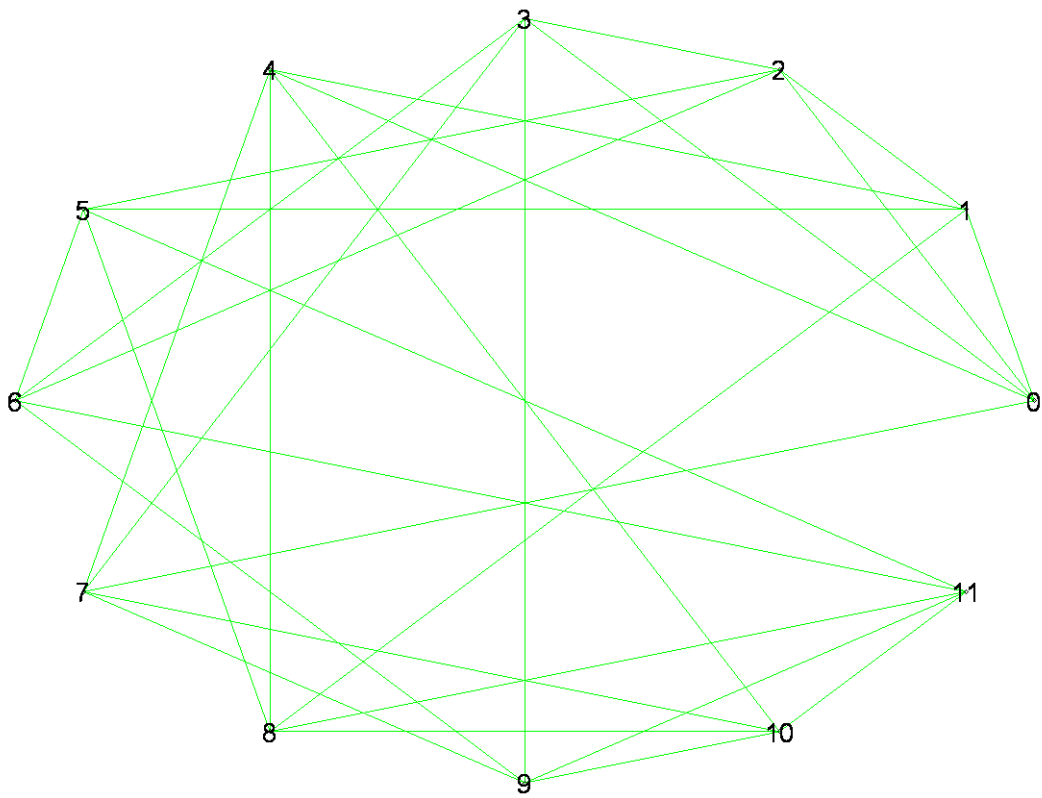
```
> K3_3:=complete(3,3);  
> draw(K3_3);
```



According to Kuratowski (circa 1930) a graph is planar if and only if it contains no subgraph that is homeomorphic to K_5 or $K_{3,3}$. Note the term "homeomorphic" is used here in a sense peculiar to graph theory: two graphs are homeomorphic if they can be obtained from the same graph by inserting vertices of degree 2 on its edges. This is a beautiful result though I have no idea how effective it is.

Maple does have a builtin planarity test, `isplane()`.

```
> G4:=icosahedron();  
> draw(G4);
```

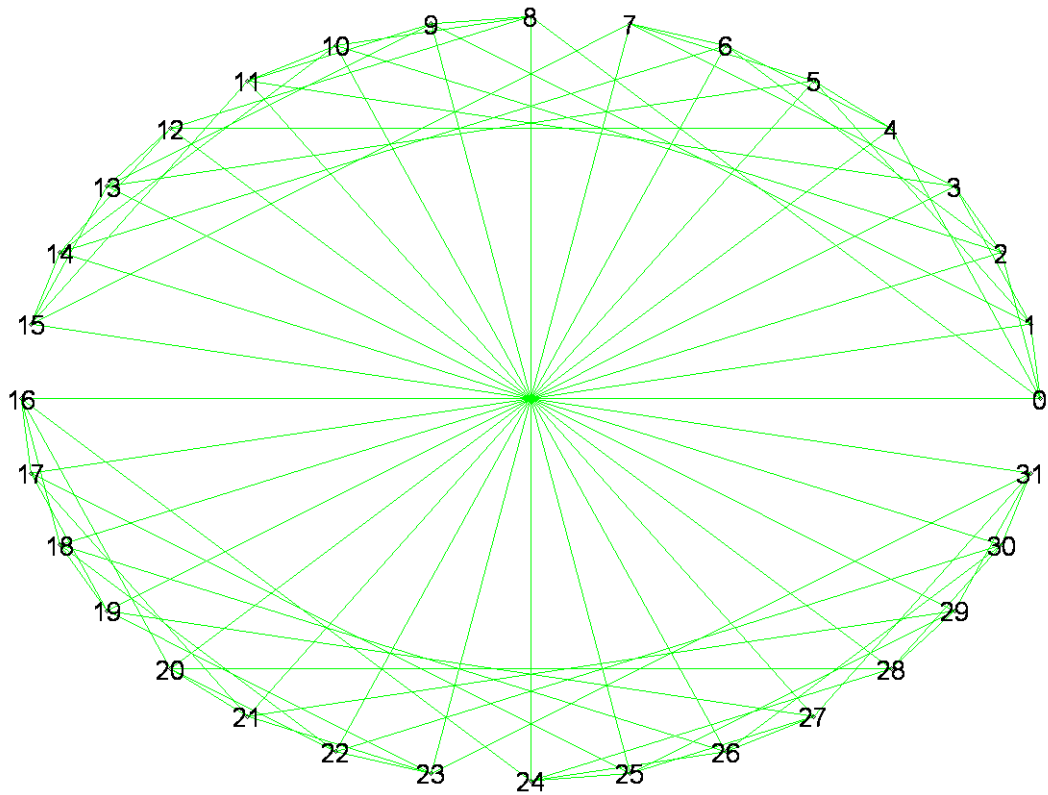


```
> isplanar(G4);
```

true

```
> G5:=cube(5);
```

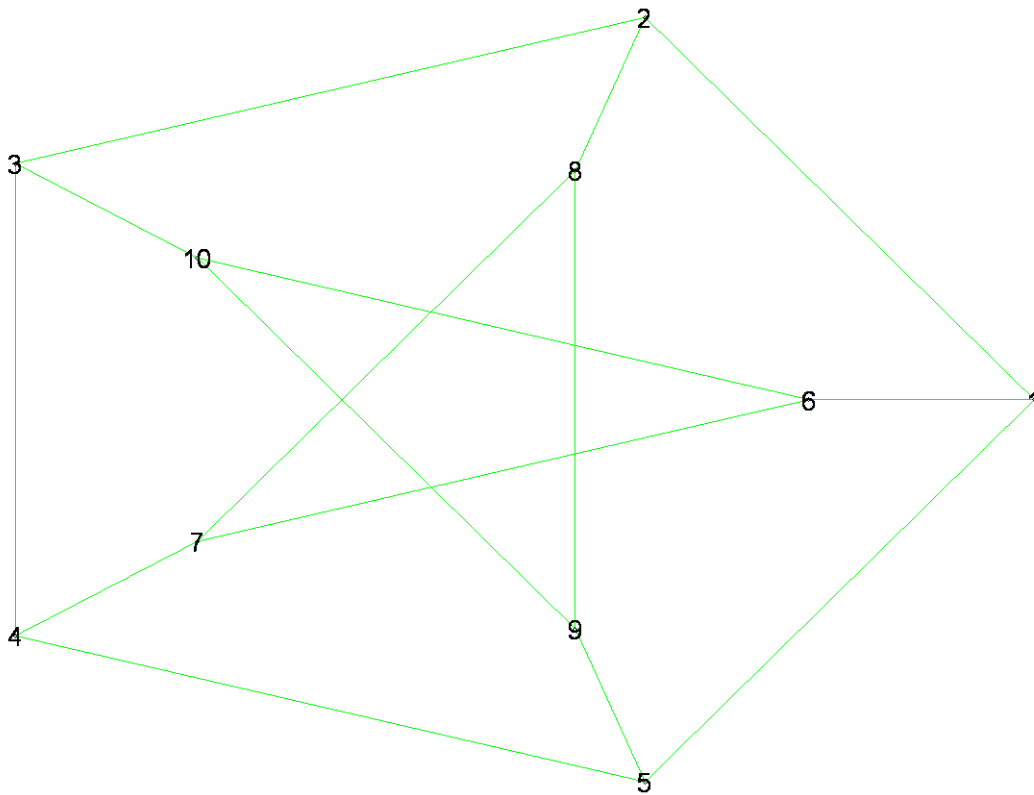
```
> draw(G5);
```



```

> isplanar(G5);
false
> isplanar(complete(4));
true
> isplanar(complete(5));
false
> isplanar(complete(6));
false
> isplanar(complete(2,3));
true
> isplanar(complete(3,3));
false
> G6:=petersen();
> draw(G6);

```



```

> isplanar(G6);
false

```

Try using Maple to do some of the problems in section 4.8 of our text (page 136).

Note Maple has more graph theory support builtin. I plan to describe more in a future version of this worksheet. If you want to know more know try consulting Maple's help.

```

>

```