

You may use one 8.5×11 inch sheet of notes. You may use a calculator.

Problem 1. (20 points). Consider an $m \times n$ rectangular grid. How many routes are there from the lower left corner to the upper right corner if we allow only travel to the right or upward? **Hint:** Each route may be associated with a string consisting of m U's and n R's where R indicates a right step and U indicates an upward step.

Problem 2. (20 points). If we toss a fair coin 20 times we can record the outcome as a sequence of H's (for heads) and T's (for tails). Clearly there are 2^{20} possible strings. What fraction of these strings contain exactly 10 H's?

Problem 3. (20 points). Let X be a set of cardinality 12 and let Y be a set of cardinality 5. The number of partitions of X into 5 non-empty subsets is $S(12, 5) = 1,379,400$. How many epimorphisms $f: X \rightarrow Y$ are there?

Problem 4. (20 points). Find the number of solutions in non-negative integers to the equation

$$x_1 + x_2 + x_3 + x_4 = 42.$$

Problem 5. (20 points). Find the number of solutions in non-negative odd integers to the equation

$$x_1 + x_2 + x_3 + x_4 = 42.$$

Problem 6. (20 points). Find the number of solutions in positive integers to the inequality

$$x_1 + x_2 + x_3 + x_4 \leq 36.$$

Problem 7. (20 points). Find the number of solutions in non-negative integers to the equation

$$x_1 + x_2 + x_3 = 10$$

if $5 \leq x_1 \leq 9$, $1 \leq x_2 \leq 6$, $x_3 \geq 2$ is odd.

Problem 8. (20 points). The generating function for a certain sequence $(a_n)_{n \geq 0}$ is

$$g(x) = \frac{1}{(1-x^2)(1-x^3)}.$$

Find a_6 .

Problem 9. (20 points). Solve the recurrence relation

$$a_n = 6a_{n-1} - 9a_{n-2}$$

with the initial conditions

$$a_0 = 2, \quad a_1 = 3.$$

Problem 10. (20 points). Solve the recurrence relation

$$a_n = 3a_{n-1} + 4n3^n$$

with the initial condition

$$a_0 = 6.$$

Use this space for scratch work.

Please do not write in the boxes to the right. They are for your grades. Do not be concerned if there are more boxes than problems.

										Letter Grade
										<input type="checkbox"/> <i>This test only</i>
										<input type="checkbox"/> <i>Cumulative</i>
1	2	3	4	5	6	7	8	9	10	Total

Note: There are 10 problems for a total of 200 points.