

You may use one 8.5×11 inch sheet of notes. You may use a calculator.

Problem 1. (20 points). How many strings are there of length 12 consisting of 3 A's, 5 B's and 4 C's?

Problem 2. (20 points). Let X be a set of cardinality 5 and let Y be a set of cardinality 12. How many monomorphisms $f: X \rightarrow Y$ are there?

Problem 3. (20 points). Find the number of solutions in positive integers to the equation

$$x_1 + x_2 + x_3 + x_4 = 42.$$

Problem 4. (20 points). Find the number of solutions in non-negative integers to the equation

$$x_1 + x_2 + x_3 + x_4 = 42$$

if $x_1 \geq 2$ and $x_2 \geq 5$.

Problem 5. (20 points). Find the number of solutions in non-negative even integers to the equation

$$x_1 + x_2 + x_3 + x_4 = 42.$$

Problem 6. (20 points). Find the number of solutions in non-negative integers to the equation

$$x_1 + x_2 + x_3 + x_4 = 42$$

if x_1 and x_2 are even and x_3 and x_4 are odd.

Problem 7. (20 points). Find the number of solutions in non-negative integers to the inequality

$$x_1 + x_2 + x_3 + x_4 \leq 36.$$

Problem 8. (20 points). Find the number of solutions in non-negative integers to the equation

$$x_1 + x_2 + x_3 + x_4 = 12$$

if $5 \leq x_1 \leq 9$, $1 \leq x_2 \leq 6$, $x_3 \geq 2$ is prime and x_4 is odd.

Problem 9. (20 points). The generating function for a certain sequence $(a_n)_{n \geq 0}$ is

$$g(x) = \frac{1}{(1-x)(1-x^4)}.$$

Find a_8 .

Problem 10. (20 points). Find the characteristic polynomial and characteristic roots of the recurrence relation

$$a_n = 7a_{n-1} - 10a_{n-2}.$$

Problem 11. (20 points). Solve the recurrence relation

$$a_n = 2a_{n-1} + 3a_{n-2}$$

with the initial conditions

$$a_0 = 8, \quad a_1 = -4.$$

Problem 12. (20 points). Solve the recurrence relation

$$a_n = 4a_{n-2}$$

with the initial conditions

$$a_0 = 2, \quad a_1 = 8.$$

Problem 13. (20 points). You deposit \$ 50 per month into an initially empty bank account. Suppose you earn $\frac{1}{2}$ percent on the account at the end of each month. Find a recurrence equation for the account balance b_n at the end of the n^{th} month. What is your balance at the end of 24 months?

Problem 14. (20 points). Solve the recurrence relation

$$a_n = 2a_{n-1} + 3a_{n-2} + n^2 3^n$$

with the initial conditions

$$a_0 = 2, \quad a_1 = 3.$$

Problem 15. (20 points). An urn contains a large number of red, white and blue marbles. How many ways are there of selecting 10 marbles from the urn if the number of red marbles must be odd and at least 3, if the number of white marbles must be even, but no more than 4, and the number of blue marbles must be prime?
