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Name:

Complex numbers were forced on Cardan in his *Ars Magna*, 1545. Cardan ignored complex roots, but even for a cubic with 3 distinct real roots, complex numbers occur in Cardan's calculation of the real roots. Cardan observes this fact, but doesn't grasp the opportunity to introduce complex numbers as objects worthy of study. We can hardly blame him. He lived at a time when negative solutions of equations were regarded as fictitious, impossible or nonsense – complex solutions were even less acceptable! Cardan did have courage enough to present negative roots, though he regarded them as meaningless – just symbols.

Complex numbers were used more and more from the 1750's on by, for example, d'Alembert, Laplace and others, and especially by Euler. However, even at this time, complex numbers were regarded as part of a technique rather than as "real" objects.

The interpretation of complex numbers as vectors in the plane is due to Wessel (1797) and Argand (1806), and as points in the plane by Gauss (1831 and earlier). These interpretations gave geometric substance to complex numbers and made them more acceptable.

In the period 1814-1846 Cauchy developed a full-blown theory of analytic functions of a complex variable. There was no longer any doubt about complex numbers being as real as any other mathematical objects.

A good reference is Morris Kline, *Mathematical Thought from Ancient to Modern Times*, Oxford 1972.

The first chapter of our text deals with complex numbers. I do not plan to lecture much on chapter 1. You should read chapter 1 – pay particular attention to the stereographic projection and the chordal metric in section 6.

Problem 1. Let $a, b \in \mathbb{C}$. Describe the set $\{\, z \in \mathbb{C} \;\mid\; |\, z - a \,| = |\, z - b \,| \,\}$.

Problem 2. Let $z_k, w_k \in \mathbb{C}$. Prove the Lagrange identity

$$\left| \sum_{k=1}^{n} z_k w_k \right|^2 = \left(\sum_{k=1}^{n} |\, z_k \,|^2 \right) \left(\sum_{k=1}^{n} |\, w_k \,|^2 \right) - \sum_{k<j} |\, z_k \overline{w}_j - z_j \overline{w}_k \,|^2 .$$

Problem 3. Show the chordal metric

$$d(z, w) = \frac{2\,|\, z - w \,|}{\left(1 + |\, z \,|^2\right)^{\frac{1}{2}} \left(1 + |\, w \,|^2\right)^{\frac{1}{2}}}$$

is not equivalent to the Euclidean distance but induces the usual topology in the plane $\mathbb{C}$. You can do the first part by producing a d–Cauchy sequence which is not Cauchy in the Euclidean metric, or directly.

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