Bent E. Petersen petersen@math.orst.edu

Use this page as cover sheet stapled onto your submission, or do all your work on this page if there is sufficient room.

Name:

Problem 1. Suppose $f(z) = \sum_{n=0}^{\infty} a_n z^n$ has positive radius of convergence R. Suppose we have r with $0 < r < R$ such that

$$\sum_{n=2}^{\infty} |\, a_n \,| n r^{n-1} < |\, a_1 \,| .$$

Show f is injective (one-to-one, monomorphic) in the disk $D(0, r)$. **Hint:** Factor $z - w$ out of $f(z) - f(w)$.

Problem 2. Show

$$f(z) = \sum_{n=1}^{\infty} \frac{1}{n^3} z^n$$

is injective in $D(0, \frac{4}{5})$.

Problem 3. Let $a \in \mathbb{C}$. Compute the radius of convergence of the power series

$$\sum_{n=0}^{\infty} a^{n^2} z^n .$$

Problem 4. Compute the radius of convergence R of the power series

$$g(z) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} z^n .$$

Show $g'(z) = (1 + z)^{-1}$ in $D(0, R)$.

Problem 5. Let a_n be a sequence of *nonzero* numbers. Show

$$\liminf_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \le \liminf_{n\to\infty} |\, a_n \,|^{\frac{1}{n}} \le \limsup_{n\to\infty} |\, a_n \,|^{\frac{1}{n}} \le \limsup_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| .$$

Problem 6. Let

$$a_n = 2^{(-1)^n - n} .$$

Compute each of the terms in the sequence of inequalities in the previous problem.

Problem 7. Compute the radius of convergence R of the power series

$$\sum_{n=0}^{\infty} 2^{(-1)^n - n} z^{2n} .$$

---

*File ref: 611s2002˙assign02.tex Wednesday April 17, 2002