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Name:

Problem 1. Let f be analytic in the right half plane $\{\, z \in \mathbb{C} \mid \mathfrak{Re}(z) > 0 \,\}$. Suppose there is a constant M such that $|\, f(z) \,| \le M$ for $\mathfrak{Re}(z) > 0$. Show that

$$|\, f'(z) \,| \le \frac{M}{\mathfrak{Re}(z)}, \quad \mathfrak{Re}(z) > 0.$$

Problem 2. Suppose $f(z)$ is entire and $\mathrm{e}^{f(z)}$ is a polynomial. Show that $f(z)$ is constant.

Problem 3. Suppose $f(z)$ is entire and there is a constant C such that

$$|\, f(z) \,| \le C\,(1 + |\, z \,|)^m, \quad z \in \mathbb{C}.$$

Show that $f(z)$ is a polynomial of degree at most m.

Problem 4. Suppose $f(z)$ is entire and there is a constant M such that

$$\mathfrak{Re}\, f(z) \le M, \quad z \in \mathbb{C}.$$

Show that $f(z)$ is constant. Similarly any of the inequalities $\mathfrak{Re}\, f(z) \ge M$, $\mathfrak{Im}\, f(z) \le M$ or $\mathfrak{Im}\, f(z) \ge M$, for all $z \in \mathbb{C}$ implies $f(z)$ is constant.

Problem 5. Suppose $f(z)$ is entire and there is a constant M such that

$$\mathfrak{Re}\, f(z) \le M \log(1 + |\, z \,|), \quad z \in \mathbb{C}.$$

Show that $f(z)$ is constant. Consider also the variations analogous to those in the previous problem. **Hint:** You may find the statements of problems 1 and 2 useful.

Problem 6. Let Ω be a connected open set in $\mathbb{C}$ and let h be analytic in $\mathbb{C}$. Suppose $h(z) \ne 0$ for each $z \in \Omega$. Then the following statements are equivalent:

1. There is a function g analytic on Ω such that $\mathrm{e}^{g(z)} = h(z)$ for each $z \in \Omega$. That is, $g(z)$ is a branch of $\log h(z)$ in Ω.
2. $\dfrac{h'(z)}{h(z)}$ has a primitive in Ω.
3. $\displaystyle\int_\gamma \frac{h'(z)}{h(z)}\, \mathrm{d}z = 0$ for each closed polygonal path γ in Ω.
4. $\displaystyle\int_\gamma \frac{h'(z)}{h(z)}\, \mathrm{d}z = 0$ for each closed rectifiable curve γ in Ω.

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