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Use this page as cover sheet stapled onto your submission, or do all your work on this page if there is sufficient room.
This final assignment is due on Wednesday of exam week - under my door or in my mail box. I may be unable to grade assignments turned in late, so please be on time.

Name:

Problem 1. Let Ω be a nonempty connected open subset of $\mathbb{C}$, let $f:\Omega \to \mathbb{C}$ be a nonconstant analytic function and suppose $f(\Omega)$ is closed. Show f maps onto $\mathbb{C}$.

Problem 2. Let f be continuous on the square $|\,\mathfrak{Re}(z)\,| \leq 2$ and $|\,\mathfrak{Im}(z)\,| \leq 2$ and analytic on the interior of the square. Suppose $|\,f(z)\,|$ is bounded by 4 on the boundary of the square and $f(0) = 0$. Show that $\left|\,f(\frac{1}{4})\,\right| \leq \frac{1}{2}$.

Problem 3. Evaluate the integral

$$\int_0^{2\pi} \mathrm{e}^{\mathrm{e}^{\mathrm{i}t}-2\mathrm{i}t}\,\mathrm{d}t.$$

Problem 4. Find the maximum of

$$\left|\,\mathrm{e}^{\mathrm{i}z}+1\,\right|,$$

$z = x + \mathrm{i}y$, on the rectangle $\frac{\pi}{2} \leq x \leq \pi$ and $0 \leq y \leq 1$.

Problem 5. Let

$$f(z) = \frac{7z-1}{(z-1)(z+1)(z-3)}.$$

Expand $f(z)$ in

1. Taylor series about $z = 0$
2. Laurent series in $1 < |\,z\,| < 3$
3. Laurent series in $|\,z\,| > 3$
4. Laurent series in $0 < |\,z-1\,| < 2$
5. Laurent series in $|\,z-1\,| > 2$.

In each case just give terms of degree -2 to terms of degree 2. Rather than evaluating integrals think carefully about the *unique* Laurent decomposition.

*File ref: 611s2002˙assign05.tex Wednesday June 12, 2002

Problem 6. Let Ω be an open subset of $\mathbb{C}$ and let f be analytic on Ω. Let $z_0, z_1, z_2, \cdots, z_n$ be distinct points in Ω. Inductively define the Newton divided differences

$$\begin{aligned} f[z_0] &= f(z_0) \\ f[z_0, z_1] &= \frac{f[z_1] - f[z_0]}{z_1 - z_0} \\ f[z_0, z_1, \cdots, z_k] &= \frac{f[z_1, \cdots, z_k] - f[z_0, \cdots, z_{k-1}]}{z_k - z_0}, \quad 2 \le k \le n \end{aligned}$$

Show

$$f[z_0, \cdots, z_k] = \frac{1}{2\pi \mathrm{i}} \int_\gamma \frac{f(w)}{(w - z_0) \cdots (w - z_k)} \, \mathrm{d}w$$

if γ is a closed rectifiable curve in $\Omega - \{z_0, \cdots, z_k\}$, null-homotopic in Ω and $\mathrm{Ind}_\gamma(z_j) = 1$ for $j = 0, 1, \cdots, k$. Deduce $f[z_0, \cdots, z_k]$ is symmetric in $z_0, \cdots, z_k$ and that $f[z_0, \cdots, z_k]$ makes sense even if $z_0, \cdots, z_k$ are not distinct. Compute $f[z_0, \cdots, z_0]$ ($k + 1$ copies of z_0 here).