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Use this page as cover sheet stapled onto your submission, or do all your work on this page if there is sufficient room.

Name:

Problem 1. Let $(a_n)_{n\geq 1}$ be a sequence of complex numbers with $a_n \neq 0$ for each $n \geq 1$. Show

$$\liminf_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| \leq \liminf_{n\to\infty} |a_n|^{1/n} \leq \limsup_{n\to\infty} |a_n|^{1/n} \leq \limsup_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|.$$

Problem 2. Let $(a_n)_{n\geq 1}$ be a sequence of complex numbers with $a_n \neq 0$ for each $n \geq 1$. If $\lim_{n\leftarrow\infty} \left|\frac{a_{n+1}}{a_n}\right|$ exists, show that

$$\lim_{n\leftarrow\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\leftarrow\infty} |a_n|^{1/n}.$$

Problem 3. Let $\sum_{n=0}^{\infty} a_n (z-z_0)^n$ have radius of convergence R. If $\lim_{n\leftarrow\infty} \left|\frac{a_n}{a_{n+1}}\right|$ exists show that

$$R = \lim_{n\leftarrow\infty} \left|\frac{a_n}{a_{n+1}}\right|.$$

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