

Turn in solutions to any two of the following drill problems. You will probably want to do all of them, but I am not insisting.

Problem 1. Find the radius of convergence

$$\sum_{n=0}^{\infty} \frac{(3 + 4i)^n}{(2i + 1)^{2n}} z^n.$$

Problem 2. Find the radius of convergence

$$\sum_{n=0}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2} z^n.$$

Problem 3. Find the radius of convergence

$$\sum_{n=1}^{\infty} \frac{n^n}{n!} z^n.$$

Problem 4. Find the radius of convergence (be careful!)

$$\sum_{n=0}^{\infty} n! z^{n!}.$$

Problem 5. Find the radius of convergence

$$\sum_{n=0}^{\infty} 2^{(-1)^n - n} z^n.$$

Problem 6. Find the radius of convergence

$$\sum_{n=0}^{\infty} \frac{(2n)!}{n!n^n} z^n.$$

You may find Stirling's formula (1730) – also given by de Moivre (1730) – useful:

$$n! = \sqrt{2\pi n} n^n e^{-n} e^{\theta} \quad \text{where} \quad \theta = \frac{\delta_n}{12n}, \quad 0 < \delta_n < 1.$$

Note δ_n increases with n and is very close to 1 already for small n .