20. Functional Analysis History

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While Fréchet did some early work on linear functionals, described in his thesis important examples of non–normable topological vector spaces and actually introduced a notion of topological affine spaces in the mid 1920's, the modern theory of topological vector spaces dates from the work of Weil [13], [12] in the late 1920's and the work of Kolmogorov [8] and von Neumann in the mid 1930's (see [11]and [9]).

Investigations of integral equations and the needs of quantum mechanics led to the creation of Hilbert space theory and more generally the theory of Banach spaces. Abstract Hilbert space was introduced by von Neumann axiomatically in 1929 though concrete examples of Hilbert spaces had been studied and applied much earlier. Banach in his 1920 thesis introduced an axiomatic theory of normed linear spaces, [1]. Early work on abstract normed spaces was also done by Wiener, Hahn and Helly (see [7]). The very important Hahn–Banach theorem was discovered by Hahn in 1927 and by Banach in 1929. Some earlier less general versions were published by Riesz and Helley. Surprisingly, Banach's 1932 text, [2], the pre–eminent text on normed spaces, still included the axioms for a vector space. Perhaps this notion was not so well–known at the time though Peano had already in 1888 given an axiomatic treatment of abstract vector space, [10] (see also [4] and [9]).

One of the characteristics of functional analysis is the study of classes of functionals, in particular the dual space, the space of all continuous linear functionals on a topological vector space. The notion of a continuous functional, that is, a continuous function whose domain consists of functions, was introduced by Volterra in the late 1880's. The word *functional*, well actually *fonctionelle*, was introduced by Hadamard around 1903. The abstract notion of a continuous functional was pioneered by Fréchet. The dual space was introduced in the abstract setting by Hahn in his 1927 paper [6] and subsequently developed by Banach and others.

In summary, the theory of normed spaces has its roots in integral equations and quantum mechanics. Apart from the weak topology, it is largely a metric theory. Banach had no real need for topology in his text, though he had to invent the notion of limits of "transfinite sequences" in order to avoid topology. The basic notions of general topology and topological vector spaces on the other hand

arose from the calculus of variations. The non–metric theory of topological vector spaces however languished until the development of Schwartz's theory of distributions in the 1940s. Of course this is a great over–simplification. For a more detailed view of the events briefly sketched above the reader is referred to Morris Kline [7], Jean Dieudonné [5] and Nicola Bourbaki [3].

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