Homework Set 4
Due 10/19/2018

1. Find the determinant of the following matrices. Show your work.

   (a) \[
   \begin{bmatrix}
   1 & 2 & 3 \\
   3 & 1 & 2 \\
   2 & 3 & 1 \\
   \end{bmatrix}
   \]

   (b) \[
   \begin{bmatrix}
   1 & 1 & 1 \\
   0 & 1 & 1 \\
   0 & 0 & 1 \\
   \end{bmatrix}
   \]

   (c) \[
   \begin{bmatrix}
   1 & 2 & 2 & 1 \\
   0 & 1 & 0 & 2 \\
   2 & 0 & 1 & 1 \\
   0 & 2 & 0 & 1 \\
   \end{bmatrix}
   \]

   (d) \[
   \begin{bmatrix}
   2 & -1 & 0 & 0 \\
   -1 & 2 & -1 & 0 \\
   0 & -1 & 2 & -1 \\
   0 & 0 & -1 & 2 \\
   \end{bmatrix}
   \]

2. To each following set of vectors, do the following:

   (1) Check if they are linearly independent.

   (2) If they are linearly dependent, write one vector as a linear combination of the others.

   (3) Find a basis for the space spanned by them.

   (a) \[v_1 = (1, 3, -1), \ v_2 = (3, 7, -7), \ v_3 = (1, 2, -3).\]

   (b) \[v_1 = (2, 1, 3), \ v_2 = (1, 0, 1), \ v_3 = (0, 2, -1), \ v_4 = (4, 2, 1).\]

   (c) \[v_1 = (3, 8, 7, -3), \ v_2 = (1, 5, 3, -1), \ v_3 = (2, -1, 2, 6), \ v_4 = (1, 4, 0, 3).\]

   (d) \[v_1 = (0, 0, 2, 2), \ v_2 = (3, 3, 0, 0), \ v_3 = (1, 1, 0, -1).\]

3. Find a basis for the subspace \(\{x \in \mathbb{R}^4 : Ax = 0\}\) of \(\mathbb{R}^4\) where

   \[
   A = \begin{bmatrix}
   1 & 2 & 3 & 1 \\
   -1 & 0 & 2 & 0 \\
   1 & 4 & 8 & 2 \\
   \end{bmatrix}
   \]

   This is called the null space of matrix \(A\).

4. Check if each following set is a subspace of \(\mathbb{R}^n\).

   (a) \(V = \{x = (x_1, x_2) : x_1 + 2x_2 = 0\}\), a line through the origin in \(\mathbb{R}^2\).

   (b) \(V = \{x = (x_1, x_2) : x_1 + x_2 = 1\}\), a line not passing through the origin in \(\mathbb{R}^2\).

   (c) \(V = \{x = (x_1, x_2) : x_2 = x_1^2\}\), a parabola in \(\mathbb{R}^2\).

   (d) \(V = \{x = (x_1, x_2, x_3) : x_1 + x_2x_3 = 0\}\) as a subset in \(\mathbb{R}^3\).