1. Determine if vector \( b = (-2, 3, 6) \) is a linear combination of the following vectors:

\[
\begin{align*}
v_1 &= (3, 1, -1), \\
v_2 &= (-1, 2, -3), \\
v_3 &= (1, -1, 2), \\
v_4 &= (2, 1, -4).
\end{align*}
\]

If yes, write \( b \) as a linear combination of these vectors.

2. To each following set of vectors, do the following:

   (1) Check if they are linearly independent.
   (2) If they are linearly dependent, write one vector as a linear combination of the others.
   (3) Find a basis for the space spanned by them. What is the dimension?

   (a) \( v_1 = (2, 3, 0), v_2 = (1, 2, -1), v_3 = (0, 4, 3) \).
   (b) \( v_1 = (1, 2, -1), v_2 = (2, 1, 3), v_3 = (-1, 0, 4), v_4 = (0, 3, 1) \).
   (c) \( v_1 = (-1, 0, 4, 3), v_2 = (2, 0, -3, 2), v_3 = (0, 1, -1, 3), v_4 = (4, -2, 1, 6) \).
   (d) \( v_1 = (0, 2, 1, -1, 1), v_2 = (1, 0, 3, 2, 0), v_3 = (-1, 1, 2, 3, 2) \).

3. Supplement additional vectors to the set \( \{v_1, v_2, v_3\} \) in Part (d) of Problem 2 to obtain a basis of \( \mathbb{R}^5 \).

4. Determine a basis and the dimension for the subspace \( \{x \in \mathbb{R}^4 : Ax = 0\} \) of \( \mathbb{R}^4 \) where

\[
A = \begin{bmatrix}
1 & 2 & 3 & 1 \\
-1 & 0 & 2 & 0 \\
1 & 4 & 8 & 2
\end{bmatrix}
\]

This space is called the \textit{null space} of matrix \( A \). Its dimension is called the \textit{nullity} of \( A \).