1. Consider the linear map
   \[ f : \mathbb{R}^3 \to \mathbb{R}^4, \quad f(x_1, x_2, x_3) = (x_1 + x_2 + x_3, 2x_1 + 4x_3, 3x_1 + 2x_2 + 4x_3, 5x_2 - 5x_3). \]
   (a) Determine \( \ker(f) \) by finding a basis. What is its dimension?
   (b) Determine \( \text{range}(f) \) by finding a basis. What is its dimension?

2. Determine all values of \( c \) such that the following linear map is bijective (i.e. both injective and surjective):
   \[ f : \mathbb{R}^3 \to \mathbb{R}^3, \quad f(x, y, z) = (x + 2y - z, x + (c + 2)y - z, x + 2y + cz). \]

3. Check if each following map is a linear map. If it is, explain why (by verifying the 2 criteria). If it is not, show how one of these criteria is violated.
   (a) \( f : \mathbb{R} \to \mathbb{R}, \quad f(x) = \sin(x). \)
   (b) \( f : \mathbb{R}^2 \to \mathbb{R}^2, \quad f(x, y) = (x^2, y^2). \)
   (c) \( f : \mathbb{R}^3 \to \mathbb{R}^2, \quad f(x, y, z) = (2x + 2y, x - z). \)

4. Let \( f : \mathbb{R}^3 \to \mathbb{R}^2 \) be a linear map such that:
   \[
   f(2, 3, 1) = (1, 0) \\
   f(1, 0, 1) = (2, -1) \\
   f(-1, -2, 0) = (-1, 1)
   \]
   (a) Find the matrix representing \( f \).
   (b) Find \( f(3, 4, 5) \).

5. Let \( f : \mathbb{R}^3 \to \mathbb{R}^2 \) be a linear map.
   (a) Can \( \ker(f) \) be 1-dimensional and \( \text{range}(f) \) be 1-dimensional? If yes, give an example for such \( f \). If not, explain why.
   (b) Can \( \ker(f) \) be 1-dimensional and \( \text{range}(f) \) be 2-dimensional? If yes, give an example for such \( f \). If not, explain why.
   Hint: Use rank-nullity theorem.

6. Consider the following vectors:
   \[
   v_1 = (1, 2, 3) \\
   v_2 = (-1, 3, -1) \\
   v_3 = (0, 2, 1)
   \]
   (a) Check if \( v_1, v_2, v_3 \) form a basis for \( \mathbb{R}^3 \).
(b) Put $S = \{v_1, v_2, v_3\}$. Find the coordinates of vector $v = (2, 1, 0)$ with respect to basis $S$.
In other words, find $c_1, c_2, c_3$ such that $v = c_1v_1 + c_2v_2 + c_3v_3$.
Note: one also denotes $[v]_S = (c_1, c_2, c_3)$. 