The matrix $A = \begin{bmatrix} 0 & 1 & 2 & -1 \\ 2 & 2 & 10 & -6 \\ 1 & 0 & 3 & -2 \end{bmatrix}$ has reduced row echelon form $B = \begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

1. What is the rank of $A$?

\( \text{rank}(A) = 2 \) (\# non-zero rows of $B = \# \text{pivot cols. of } B$)

2. Determine a basis of the column space of $A$. What is its dimension?

\( \dim \text{C}(A) = 2 \)

3. Determine a basis of the row space of $A$. What is its dimension?

\( \dim \text{R}(A) = 2 \)
4. Supplement more vectors to the basis of the row space which you obtain in Part 3 to get a basis for $\mathbb{R}^4$.

\[
(2 \text{pt}) \quad \text{add these vectors:} \quad (0, 0, 1, 0), \quad (0, 0, 1, 1).
\]

5. Determine a basis for the null space of $A$. What is its dimension? (In other words, what is the nullity of $A$?)

\[
(3 \text{pt}) \quad B = \begin{bmatrix}
1 & 0 & 3 & -2 \\
0 & 1 & 2 & -1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[\begin{align*}
x_4 &= t, \quad x_3 = s \\
\text{From the second row of } B: \quad &2x_2 + 2x_3 - x_4 = 0 \\
&\implies x_2 = -2s + t
\end{align*}\]

\[\text{From the first row:} \quad x_1 + 3x_3 - 2x_4 = 0 \\
&\implies x_1 = -3s + 2t
\]

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix} = \begin{bmatrix}
-3s + 2t \\
-2s + t \\
s \\
t
\end{bmatrix} = s \begin{bmatrix}
-3 \\
-2 \\
1 \\
0
\end{bmatrix} + t \begin{bmatrix}
2 \\
1 \\
0 \\
1
\end{bmatrix}
\]

Basis: \[\left\{ \begin{bmatrix}
-3 \\
-2 \\
1 \\
0
\end{bmatrix}, \begin{bmatrix}
2 \\
1 \\
0 \\
1
\end{bmatrix} \right\}\]

\[\dim \ N(A) = 2.\]